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EXPERIMENTAL INSTRUMENTS AND TECHNIQUES

Experimental Demonstration of the Sensitivity of an Optically Pumped Quantum Magnetometer

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Abstract—The sensitivity σ of an optically pumped quantum magnetometer is measured directly by comparing the output frequencies of ^{85}Rb and ^{87}Rb magnetometers integrated in a single two-isotope cell. The result $\sigma_{87} = 59.6 \pm 6$ fT Hz$^{-1/2}$ agrees well with the value $\sigma_{87}^{\text{light}} = 53.7 \pm 2$ fT Hz$^{-1/2}$ obtained indirectly by measuring the ratio of the resonance steepness to the shot noise of light. *© 2004 MAIK "Nauka/Interperiodica".*

1. INTRODUCTION

In the last few decades, a great step forward has been made in the field of quantum magnetometry [1–4]. New types of optically pumped quantum magnetometers (OPQMs) that demonstrate a low-field short-term sensitivity on the order of several tenths of a femtotesla have been designed [5]. The parameters of conventional OPQMs that operate in the geophysical range of magnetic fields (20–80 μT) have also been improved. The most vivid example is a potassium-vapor OPQM, which combines a sensitivity of 10 fT Hz$^{-1/2}$, a long-term stability in fields on the order of 10 pT, and an absolute accuracy of 0.1 nT [2]. Similarly high sensitivities may be reached in still lower fields (<15 μT) with Cs and Rb magnetometers operating on the unresolved Zeeman structure. Magnetometers with a high sensitivity in this range are needed, for example, for the fundamental experiment on detecting the permanent dipole moment of a neutron [6].

The sensitivity of OPQMs is usually estimated indirectly by measuring the resonance steepness-to-noise ratio [7]. This approach is based on the assumption that a minimal change in the magnetic field detectable by a magnetometer in a given frequency band (for example, 1 Hz) can be expressed as

$$\begin{aligned}\delta B_{\min} &= (1/\gamma)N/(dS/df)|_{f=f_0}\\ &= (1/\gamma)kN/(S/\Gamma).\end{aligned} \qquad (1)$$

Here, γ is the gyromagnetic ratio, $f = f_0$ is the magnetic resonance frequency, $k \approx 1$ is the resonance form factor, Γ is the resonance line width, N is the rms noise level measured in the same frequency band, and S is the signal amplitude. The parameters k, S, Γ, and N can be measured directly in experiments; otherwise, the experimentalist can measure the resonance steepness $S' = (dS/df)|_{f=f_0}$ at the center of the resonance line and the noise level away from the resonance and then calculate the sensitivity as $\delta B_{\min} = (1/\gamma)/N/S'$.

The latter approach, however, implies that the OPQM noise depends on the shot noise of light. This, in turn, means that (i) the noise inside and outside the resonance line is the same and (ii) the noise in the closed feedback loop is also defined by the shot noise of light. In other words, this approach implies that atomic fluctuations are negligible compared with the shot noise (which is the case for OPQMs) and that any type of technical noise may be reduced to the shot noise of light.

These assumptions, while quite plausible, have never been substantiated experimentally. Difficulties associated with experiments in this field are the following: to measure the inherent noise of the magnetometer, which is responsible for the ultimate sensitivity, it is necessary to eliminate the contribution of magnetic field variation to the measurand (the frequency of double radiooptical resonance in fields on the order of several femtoteslas). To stabilize the field with such an accuracy is a challenge at least in the geomagnetic range.

Under normal conditions, the noise of the terrestrial magnetic field exceeds the OPQM noise by four or five orders of magnitude. Moreover, the magnetic field gradient variation cannot, as a rule, be suppressed below several hundreds of fT/Hz$_1$/2 per meter.

For many fundamental reasons, the magnetic field variation in a set of Helmholtz rings and/or multilayer magnetic screens is also difficult to suppress below a certain level. It is obvious that any system suppressing magnetic field variations based on readings of a single detector may reduce the variation only to the sensitivity level of this detector and only near its location. Suppression of first- or second-order magnetic field gradients calls for much more sophisticated multidetector equipment. There are also other fundamental limita-

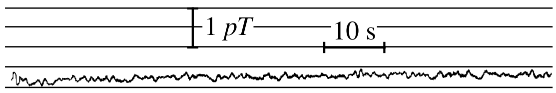

Fig. 1. Magnetic field gradient recorded under ultraquiescent magnetic conditions (Georgina island, Canada) when two potassium OPQMs were compared (the cell volume 1800 cm^3).

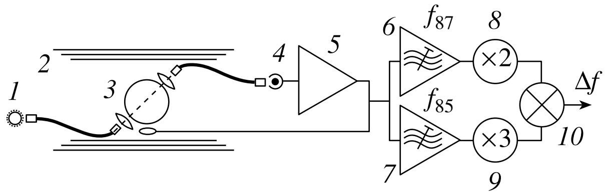

Fig. 2. Schematic of the experimental setup: (*1*) lamp containing the mixture of ^{87}Rb and ^{85}Rb isotopes; (*2*) magnetic screen; (*3*) cell containing the mixture of ^{87}Rb and ^{85}Rb isotopes; (*4*) photodetector; (*5*) wide-band amplifier; (*6*, *7*) narrow-band amplifiers tuned to the self-oscillation frequencies of ^{87}Rb and ^{85}Rb, respectively; (*8*, *9*) frequency multipliers; and (*10*) balanced mixer. The intermediate and terminal narrow-band amplifiers, microwave lamp-exciting generator, temperature stabilization loops of the lamp and cell, and magnetic field stabilization loops are omitted.

tions, such as thermal current noise in the metallic screen, etc.

Thus, the only way to directly measure the sensitivity of the magnetometer is to compare its readings with those of a device with the same or higher sensitivity provided that a change in the gradient between the locations of two devices does not exceed the sensitivity expected. In the case of OPQMs, this is difficult to accomplish: two similar OPQMs cannot be placed closer than 50 cm, since crosstalk between rf channels may be a problem.

That is why the sensitivity as high as 100 fT $Hz^{-1/2}$ or better calculated from the resonance steepness-to-noise ratio measured is so difficult to demonstrate experimentally (unless the field is ultralow [5]). To our knowledge, the OPQM highest sensitivity 60 fT $Hz^{-1/2}$ in the terrestrial field was demonstrated as a result of cooperation between the GEM Systems Co., Inc. (Toronto, Canada) and the authors of this work (Vavilov State Optical Institute, St. Petersburg, Russia). Such a high value was obtained under extremely quiescent magnetic conditions by comparing two potassium OPQMs with a cell volume of 1800 cm^3 (Fig. 1). However, even in this case, the actual relative noise of the two magnetometers was higher than estimated by a factor of 8 to 20. It therefore becomes clear why metrologists and geophysicists are sceptical about high sensitivities thus obtained and go on insisting on direct measurements.

In this work, we report direct measurements of the OPQM sensitivity at a level of several fT $Hz^{-1/2}$ in a finite (nonzero) magnetic field. Our data are in good agreement with those obtained from the resonance steepness-to-shot noise ratio.

2. EXPERIMENTAL

We eliminated the effect of magnetic field gradient variation by integrating two rubidium-isotope magnetometers in a single cell and taking the difference between their readings. The experimental setup is shown in Fig. 2. In a laboratory screen exposed to a field B = 1170 nT, two single-beam self-oscillatory magnetometers were made. They operate on the unresolved structure of lines in the F = 2 state of ^{87}Rb and ^{85}Rb isotopes, are pumped with an rf gas-discharge lamp, have a mutual feedback loop, and share a cell with walls of diameter 70 mm covered by an antirelaxation coating. The cell contains a drop of a rubidium isotope mixture in the proportion 1 : 1. Pumping was accomplished with circularly polarized light from the ^{87}Rb D_1 and ^{85}Rb D_1 lines of the spectroscopic lamp filled with the same isotope mixture. Near-optimal conditions for self-oscillation at both spectral lines were set by appropriately selecting the signal phase and amplifier gain. The setup was placed in the trilayer magnetic screen, and the signal from one magnetometer was used to stabilize the magnetic field 1170 nT inside the screen.

The use of the mutual cell made it possible to completely eliminate the gradient variation problem, since atoms of both isotopes in the given configuration are subjected to the same field averaged over the cell volume. Such a configuration of the magnetometers cannot provide a record sensitivity that is comparable, e.g., to the sensitivity of potassium magnetometers (this issue will be touched upon below). Here, we would like to stress that our primary goal was to experimentally check the validity of determining the sensitivity by measuring the steepness-to-noise ratio, rather than to achieve a record sensitivity.

The gyromagnetic ratios for ^{87}Rb (F = 2) and ^{85}Rb (F = 3) are given by

$$\gamma = \mu_B(g_J - 2Ig_1)/(2I+1), \qquad (2)$$

For ^{85}Rb, I = 5/2, gJ = 2.002331, and g_1 = 0.294745 × 10^{-3}; for ^{87}Rb, I = 3/2, gJ = 2.002331, and g_1 = 0.998823 × 10^{-3} [8]. The value of μ_B equals 13.99624624(56) Hz/nT [9]. Then, γ_{85} = 4.667415 Hz/nT and γ_{87} = 6.995795 Hz/nT.

Using the proximity of the ratio γ_{87}/γ_{85} to the rational number 3/2, we devised a simple measuring scheme that effectively suppresses residual magnetic field fluctuations. To this end, the self-oscillation signal from the two isotopes that was detected by the same detector and amplified by the same amplifier was applied to two resonant amplifiers, each separating the self-oscillation signal at the frequencies f_{85} and f_{87} corresponding to the isotopes. Simultaneously, the self-oscillation signal of ^{85}Rb was used to stabilize the field in the cell volume with a precision reference oscillator. Then, the frequency f_{85} = 5465 Hz was multiplied by three and the frequency f_{87} = 8192, by two. In this way, the field-to-frequency conversion coefficients became equal to each other up to the contribution from the nuclear magnetic moment $\Delta f_N = 8 \times 10^{-4} f_{85}$. Next, the difference frequency $\Delta f = (3f_{85} - 2f_{87}) = (3\gamma_{85} - 2\gamma_{87})B = \mu_B(3/2g_1^{87} - 5/2g_1^{85})B = 12.47$ was separated with a balanced mixer and measured in the field B = 1170 nT (Δf = 12.47 Hz). The frequency Δf is other than zero only due to the contribution from the nuclear moments (Δf may be slightly different from the above value because of imperfect phase tuning in the self-oscillation circuits), and its dependence on the magnetic field is three orders of magnitude weaker than the field dependence of f_{85} and f_{87}. Thus, the effect of residual magnetic field variation on the difference frequency is suppressed by a factor of 1000 with this measuring scheme. At the same time, crosstalk between the rf channels of the magnetometers is completely eliminated.

To evaluate the suppression of magnetic field fluctuations, we, along with difference frequency measurements, recorded the light intensities at the entrance to and the exit from the cell. In such a way, we could trace the contribution of a frequency shift varying as the light intensity to the output frequency of the magnetometer.

The signal at the difference frequency Δf was recorded with a 14-bit analog-to-digital converter (ADC). The value of Δf averaged over 1 s was calculated from 512 counts by using fast Fourier transformation.

The value of the magnetic field in the screen (B = 1170 nT) was chosen from the following considerations. On the one hand, the field must be taken sufficiently low so that the Zeeman split of the Rb spectrum and the line broadening due to magnetic field nonuniformity in the screen contribute insignificantly to the resonance line width. On the other hand, the measurand Δf, which varies as the magnetic field, must be sufficiently high in order that be measured with a relative accuracy of 10^{-5} for a time of about 1 s. Note that an accuracy of 10^{-5}, with which the difference frequency was measured, corresponds to an accuracy of 10^{-8} in measuring the carrier frequency, since the difference frequency was three orders of magnitude lower than both carriers.

It is significant that, despite both magnetic resonance signals were detected by the same photodetector, the noise levels in the two magnetometers are not mutually correlated. The self-oscillation circuit of either of the magnetometers separates out from the wide-band noise spectrum a narrow spectral band of width Γ (Γ = 1–4 Hz is the magnetic resonance width) around the resonance frequency f_{85} or f_{87}. Since these frequency bands are 2.7 kHz apart, the shot noise levels in them are mutually independent. However, an extra noise is present in each of the channels, because the radiation of one isotope is parasitic for the radiation of the other. In our case, this effect increases the radiation intensity roughly twofold, increasing the rms noise in each of the channels by a factor of $\sqrt{2}$.

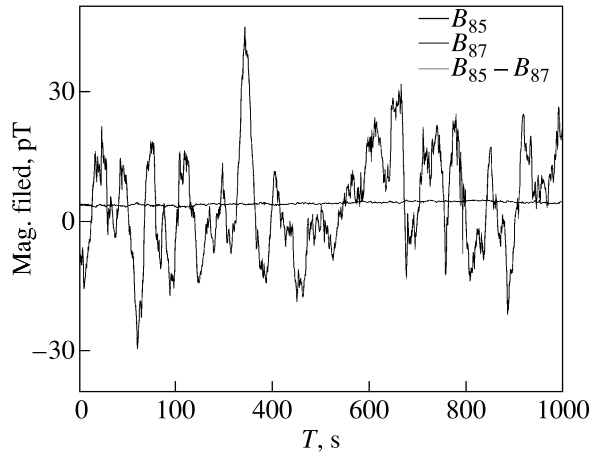

Fig. 3. Magnetic field (B) records in the ^{87}Rb and ^{85}Rb magnetometers and the difference between the records.

The measurements were carried out at a cell temperature of 30°C and a total photocurrent of 21 μA. To eliminate the residual dependence of the self-oscillation frequency on the radiation intensity and cell temperature, we stabilized the temperature of the lamp projection, which contained the drop of the metal, and the cell temperature. The cell temperature was kept constant accurate to 2°C. The dark linewidth was 1 Hz; the operating linewidth, 3.5 Hz (Fig. 4).

A series of difference frequency measurements were made over given times of up to 6000 s, and then the difference frequency variance $\sigma_{\Delta f}$ was estimated. To relate this variance to the variances of the ^{87}Rb and ^{85}Rb channels, we must know the ratio of the channel weights, σ_{87}/σ_{85}, and the inherent noise N_{MS} of the measuring system:

$$\sigma_{\Delta f}^2 = \sigma_{87}^2 + \sigma_{85}^2 + N_{MS}^2. \quad (3)$$

To measure the ratio of the channel weights, we first measured the short-term sensitivity in the broken feedback loop of each of the channel for 1 s. Resonance was excited in a stabilized field, the variance component of the signal was detected with a synchronous detector, and the response of the synchronous detector to a small (compared with the resonance width) frequency offset from the resonance was recorded. In other words, the resonance steepness $S' = \Delta S/\Delta f$ and the noise signal amplitude were measured. The ratio of the variational

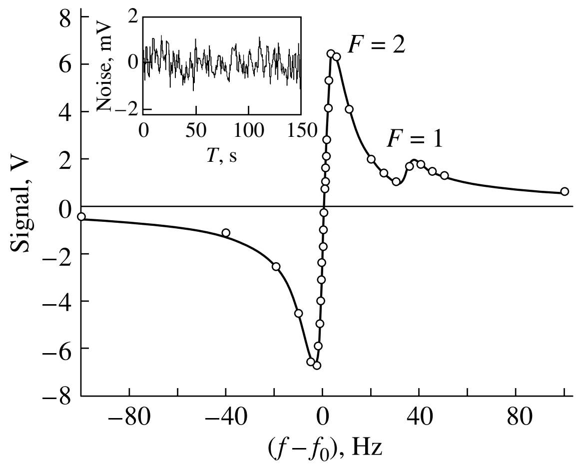

Fig. 4. Resonance line of ^{87}Rb versus the shot noise of the photocurrent ($\tau = 1$ s). Γ(HWHM) = 3.74 Hz, f_0 = 8192 Hz, dS/df = 3.33 V/Hz, $\sigma_{87}^{\text{light}}$ = 21.5 fT s$^{1/2}$ = 53.7 ± 2 fT Hz$^{-1/2}$.

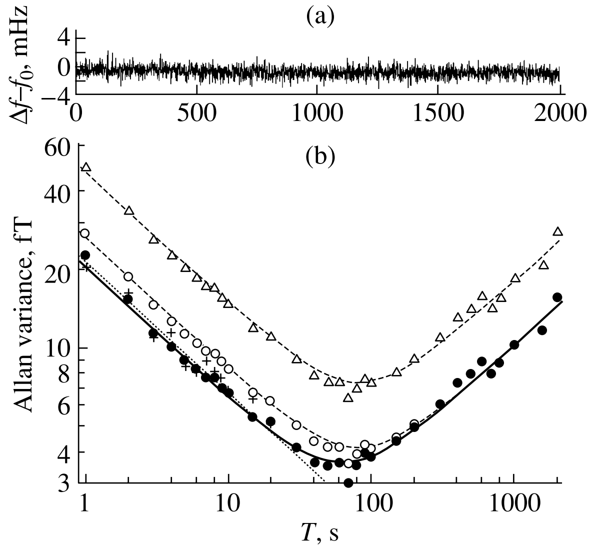

Fig. 5. (a) Record of the difference frequency $\Delta f = (3f_{85} - 2f_{87})$, f_0 = 12.868 Hz; (b) (○) Allan variance of the difference frequency Δf, (o) Allan variance of the frequency f_{87} (including the measuring system noise), (●) Allan variance of the frequency f_{87} (the measuring system noise subtracted), and (+) sensitivity of the ^{87}Rb channel that is found from the resonance steepness-to-shot noise ratio.

sensitivities of the channels was found to be $\delta B_{\text{min}}^{85}/\delta B_{\text{min}}^{87}$ = 1.44 ± 0.08. Since the amplitudes, linewidths, and spectral densities of the noise for both signals turned out to coincide within 5%, one can argue that the ratio of the variational sensitivities is consistent with the theoretical predictions and depends on the difference in the gyromagnetic ratios of the Rb isotopes (γ_{87}/γ_{85} = 1.499).

Using the ratio measured and formula (3), we can relate the difference frequency variance $\sigma_{\Delta f}$ to the frequency variances of the ^{87}Rb and ^{85}Rb channels: $\sigma_{87} = (\sigma_{\Delta f}^2 - N_{\text{MS}}^2)/178$ and $\sigma_{85} = (\sigma_{\Delta f}^2 - N_{\text{MS}}^2)/1.26$.

The fluctuations in the frequency synthesizer and frequency meter were measured by application of signals from G3-110 reference frequency generators to their inputs. The sensitivity threshold of the ADC was found to be 20 fT Hz$^{-1/2}$ (including the noise component due to fast Fourier transformation). When estimating the inherent noise of the entire measuring channel, including the frequency synthesizer, by the same method (i.e., by application of the signals from the reference generators instead of the real signal), we faced the problem of the inherent noise of the generators. We were not able to improve the measurement accuracy: this would require reference generators with an inherent noise of lower than 10^{-8} Hz$^{-1/2}$. The value measured with the G3-110, 105 ± 20 fT Hz$^{-1/2}$, should, therefore, be viewed as the upper level of the fluctuations inherent in the measuring scheme (the lower one is thus 20 fT Hz$^{-1/2}$).

We tried to narrow the range of estimates, assuming that, under a constant level of illumination, the inherent noise of a magnetometer is inversely proportional to the resonance steepness (as follows from (1)), while the noise N_{MS} of the measuring system is additive and independent of the resonance parameters. An attempt was made to isolate this resonance-independent component, varying (under constant illumination) the rf field amplitude and tracing the dependence of the resonance steepness S' on the channel noise σ_{87} measured directly. Approximating this dependence by the model formula $\sigma_{87}(S') = [(A/S')^2 + N_{\text{MS}}^2]^{1/2}$, we obtained N_{MS} = 71 ± 20 fT Hz$^{-1/2}$. Such an estimation is certainly based on the same assumption as the method of finding the sensitivity from the steepness-to-noise ratio. Therefore, this estimate, strictly speaking, may be used if the validity of this method is proved. It will be shown below that this method provides good agreement with experimental data even without taking into account the noise of the measuring system.

An example of calculating the ultimate sensitivity of the magnetometer from the steepness-to-shot noise ratio is given in Fig. 4. Figure 5 shows the results of direct measurements: the time series of the difference frequency and the Allan diagram for the difference frequency Δf. The symbols are data points; the curves approximate the dependence of the noise on the time of measurement.

From Fig. 5, it follows that the sensitivity of our magnetometers depends on the shot noise over averaging times shorter than 50 s. The Allan variance reaches a maximum, ~4 fT, over times 60 to 100 s and then grows as $T^{1/2}$: $\sigma_{87}^{\text{long}} = (0.33 \text{ fT s}^{1/2})T^{1/2}$.

The Allan variance of the difference frequency Δf in the time interval 1–50 s is $\sigma_{\Delta f}$ = 118.4 ± 4 fT Hz$^{-1/2}$. Converting it to the frequency variance of the ^{87}Rb channel, we get σ_{87} = 66.4 ± 2 fT Hz$^{-1/2}$. With regard to our estimate of the noise N_{MS} of the measuring system, the corrected value is σ_{87}^{corr} = 51 ± 9 fT Hz$^{-1/2}$ (Fig. 4). Thus, the procedure of determining the sensitivity of a magnetometer from the ratio of the resonance steepness to the shot noise of radiation has been substantiated experimentally for the first time.

Note in conclusion that the sensitivity demonstrated in this work is lower than the ultimate value expected for OPQMs. The basic factors limiting the sensitivity in our case are as follows.

(i) The cell used in the experiments was relatively small (180 cm_3) to avoid line broadening due to magnetic field nonuniformity in the screen.

(ii) As was mentioned above, the radiation of one isotope is parasitic for the radiation of the other in a two-isotope configuration. Because of this, the sensitivity was reduced by a factor of $\sqrt{2}$.

(iii) In a two-isotope configuration, the line of one isotope is excessively broadened, since atoms of one isotope experience spin-exchange collisions with atoms of the other isotope.

(iv) In the case of rubidium, the efficiency of optical detection of optical orientation is roughly twice as low as in the case of potassium, because the hyperfine split of the ground and excited states in the former case is much higher.

Our experimental verification of the procedure of finding the sensitivity from the resonance steepness-to-short noise ratio supports its validity at sensitivity levels achieved today. It seems quite possible that new noise sources, for example, the magnetization noise of the working medium, will show up when the noise level is reduced further.

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