Maximizing the quality factor for 0–0 resonance in optically pumped rubidium vapor

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1. Radiooptical resonance involving the hyperfine structure of ^{87}Rb atoms in the ground state has held the interest of engineers and physicists ever since efficient optical pumping of Rb by isotopically filtered light was first demonstrated in Ref. 1. Much of this interest derives from the usefulness of optical pumping in Rb frequency and time standards.[2,3] Much work has been done on the factors that limit the stability of Rb standards and on ways to minimize them. Although advances continue to be made along these lines, some fundamental questions remain unanswered about how the optical pumping conditions influence the frequency stability of the Rb vapor to lowest order.

Our point of departure in this note is the pioneering work in Ref. 4 and the subsequent paper Ref. 5, where an attempt was made to optimize the Rb vapor density and the power ratio of the pumping and microwave fields so as to make the frequency discrimination as sharp as possible. However, the noise in the recording channel was neglected, even though (together with the slope of the frequency discrimination curve) it determines the resolution; a correct optimization procedure must therefore use other criteria, such as those based on the quality factor Φ (Refs. 6-8). In this approach, Φ is maximized when the pumping intensity and the rf power take values which are uniquely determined by a single physical parameter, the time required for the excited Rb atoms to relax in the dark. We will show that the optimum pumping intensity found by this method is substantially lower than the values used in previous treatments and widely employed in practice. It is thus possible to stabilize the frequency of the resonance

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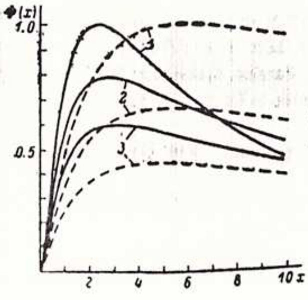

FIG. 1. Quality factor for 0–0 resonance as a function of the pump intensity.

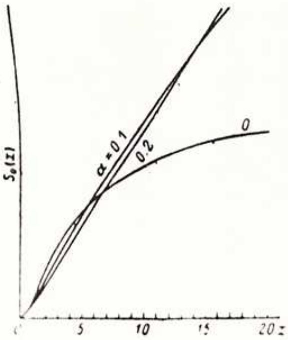

FIG. 2. Signal strength for 0–0 resonance versus the optical pump intensity.

peak without sacrificing high Φ, because the destabilizing light-induced shifts are proportional to the pump intensity.

2. In Rb frequency standards, the ^{87}Rb is pumped by light which has first been filtered in ^{85}Rb vapor so that its wavelength coincides with the D_1 and D_2 lines of ^{87}Rb. This isotopic filtering suppresses the long-wave hyperfine components in the D_1 and D_2 lines, so that the filtered light preferentially excites ^{87}Rb atoms from the F=1 level of the $5^2S_{1/2}$ ground state to the $5^2P_{1/2,3/2}$ state. The selective excitation of the F=1 level produces a population inversion with the F=2 level of the ground state as the upper level, so that the ^{87}Rb vapor becomes transparent to the pumping light. The microwave field induces a resonance between the F=1, $m_F=0$ and the F=2, $m_F=0$ sublevels (referred to as 0–0 resonance), which increases the optical absorption of the pumping light in the cell. This can be exploited to generate an error signal for regulating the microwave frequency.

An analytical expression relating the error signal to the detuning from resonance can readily be found under the following assumptions: 1) the pump intensity and spectrum (averaged over the volume of the cell) remain unchanged upon saturation of the 0–0 transition; 2) the thermal relaxation time for the populations are identical for all eight magnetic sublevels of the ground state. The change S in the intensity of the transmitted pumping light caused by the microwave field is then given by

$$S = kq\frac{(a_1-a_2)^2}{(5a_1+3a_2)\left(5+\frac{a_2}{a_1}+2\frac{a_1}{a_2}\right)}\frac{H^2}{\Delta^2+\Gamma^2+H^2},$$

$$H^2 = \frac{2\left(\frac{\omega_1}{q}\right)^2\Gamma\left(5+\frac{a_2}{a_1}+2\frac{a_1}{a_2}\right)}{5a_1+3a_2},$$

where k is a constant; $a_i = w_i/q+1$; w_i is the rate of optical excitation for atoms at level F=i (i=1, 2); q is the thermal relaxation rate; $\Delta=\Delta\omega/q$; $\Delta\omega$ is the detuning from the 0–0 resonance, and ω_1 is the Rabi frequency for the 0–0 transition in the microwave field; $\Gamma=\gamma/q$; $\gamma=[(w_1+w_2)/2+q]$ is the coherence relaxation rate. As in Refs. 4 and 5, we assume that when no light beam is present, the coherence relaxes at the same rate as the populations.

The signal S is Lorentzian. The resolution of discriminators tuned to 0–0 resonance is known to be proportional to the quality factor Φ, which is defined as the signal strength S(0) at $\Delta=0$ divided by the product of the noise amplitude and the width of the resonance at half-maximum. The quality factor is a maximum when the microwave field is such that the resonance line is $\sqrt{3}$ times broader than its natural width Γ, i.e., when $H^2=2\Gamma^2$. The corresponding maximum value Φ_{opt} depends on the intensity and spectrum of the pumping light; to calculate this dependence, one must allow for the noise in the signal channel. For a properly designed recording circuit, the noise is determined by the shot noise in the photocurrent, i.e., it is proportional to the square root of the pump intensity, or to $(w_1+w_2)^{1/2}$.[1)] We then obtain the expression

$$\Phi_{opt}(x) = k'q^{-1/2}\frac{(1-\alpha)^2}{(1+\alpha)^{1/2}}x^{3/2}$$
$$\times\left\{[2+x(1+\alpha)][8+x(5+3\alpha)]\left[5+\frac{1+\alpha x}{1+x}+2\frac{1+x}{1+\alpha x}\right]\right\}^{-1},$$

for Φ_{opt}, where k' is a new constant, $x=w_1/q$, and $\alpha=w_2/w_1$.

Figure 1 shows how Φ_{opt} depends on the dimensionless intensity x (solid curves) for three values $\alpha=0$ (1), 0.1 (2), and 0.2 (3), together with the results of a similar calculation for a model two-level system[7] (dashed curves). The curves for $\alpha=0$ are normalized by their maximum value.

3. Figure 1 shows that Φ passes through a maximum as x increases, and the position x_{max} of the maxima for the actual eight-level system is much less than x_{max} for the two-level model. This has a simple physical explanation – the Rb atom has four sublevels F=2, $m_F=\pm1, \pm2$, whose populations increase for large x; this is accompanied by depopulation of the two sublevels $m_F=0$ involved in the working transition. Because this depopulation does not occur in the model two-level system, higher intensities x are required before Φ starts to decrease due to optical broadening.

As was mentioned above, the previous work[4,5] was concerned primarily with maximizing the steepness of the frequency discrimination as a function of the optical density of the Rb vapor, and the pump intensity grid employed in the calculations was chosen to correspond to the intensities typical in applications. For example, the calculation in Ref. 4 was carried out for x=10 and $\alpha=0$ and 0.25.

We now see that better results can be obtained at much lower pump intensities. According to Fig. 1, Φ is greatest when x=2.3 ($\alpha=0$), x=2.9 ($\alpha=0.1$), and x=3.1 ($\alpha=0.2$). Because Φ falls off only very slowly for x above the optimum values it might seem harmless to employ higher intensities, and indeed it may be technically more convenient to do so, because the absolute power of the optical signals increases monotonically (Fig. 2 plots S(0) as a function of x). However, although this has little effect on the resolution of the discriminator, pump intensities that are higher than necessary increase the fluctuations in the position of the resonance because the light-induced shifts are proportional to the intensity. From this standpoint it is preferable to take x slightly less than the optimum value (within the region where Φ decreases only slightly as x decreases). Thus for $x = 1$, Φ is just 23% less than Φ_{opt} (for $\alpha = 0$). In addition, as was noted in Ref. 7, most of the noise in the signal channel is due to rapid phase (frequency) fluctuations in the microwave field. In typical quartz oscillators and frequency synthesizers, this noise is roughly ten times greater than the photocurrent shot noise. This implies that the pump intensity can be decreased still further for additional long-term stabilization without any sacrifice in discriminator resolution.

[1])This is valid when the microwave field is turned off, or if there are no fluctuations in the phase of the microwave field, see below.

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