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GEOMETRICAL AND APPLIED OPTICS

A New Method of Absolute Measurement of the Three Components of the Magnetic Field

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Abstract—A method of simultaneous measurement of the three components of the Earth's magnetic field vector using an optically pumped M_x magnetometer placed into a symmetric system of magnetic coils is proposed and justified mathematically. The method shows a high absolute accuracy (of the order of 0.1 nT for a measuring time of 0.1 s). The short-term sensitivity of the measurements is determined by the sensitivity of the M_x magnetometer.

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INTRODUCTION

In this paper, a new solution is proposed for the invariably topical problem of simultaneous measurement of all three components of the Earth's magnetic field vector. The solution is characterized by a high absolute accuracy and a fast response.

Metrologists are well aware of the fact that any component of the magnetic field can be measured with a high accuracy by measuring the field modulus provided that the field components orthogonal to the one measured are preliminarily compensated using a system of magnetic coils. In these measurements, it is not necessary to compensate the orthogonal components with a high accuracy because, in conformity with the law of vector addition, the contribution of small transverse components to the field modulus, in the presence of a large uncompensated component, is suppressed by several orders of magnitude.

However, it is difficult to use this principle for development of measuring instruments because completely compensating the two field components needed to measure the third one implies the creation of rather strong compensating magnetic fields. As a result, the procedures of measuring all three field components using this method should be separated either in space or in time. In the first case, three systems of magnetic coils should be spaced by sufficiently large distances (several meters or tens of meters) to avoid mutual interference. The second case requires the compensating magnetic fields to be rapidly switched in a single system of magnetic coils; it also implies the presence of a fast sensor for measuring the magnetic field modulus, capable of tracking jumps of the magnetic field modulus occurring on passage from measuring one component to measuring another one. For this reason, three-component devices use, as a rule, another principle: the magnetic field modulus meter is placed into a system of magnetic coils, which is intended, however, not for compensation of transverse components, but rather for creation of successive calibrated deviations of the magnetic field vector from its initial direction [1–8]. The magnetic field modulus, in such devices, is measured using either proton magnetometers [4, 6] or optically pumped magnetometers [5, 7–9]. The magnetic field direction may be switched, in these devices, either discretely or continuously (a device with the total magnetic field vector continuously rotating around that of the Earth's magnetic field is described in [9]).

A common drawback of these systems is that the measurements are not absolute, in the sense that the result of the measurement always contains a contribution from the combination of calibrated fields, which deviate the field vector from its initial direction. This contribution, being comparable, in order of magnitude, with the value of the measured field itself, should be taken into account in processing the results of measurements. This contribution, however, depends on the configuration of the coils and on their currents and, therefore, on temperature, humidity, and a number of other factors.

In this paper, a method of absolute and simultaneous measurement of all three components of the magnetic field vector using a system of magnetic coils and a sensor for measuring the magnetic field modulus is proposed. It is assumed that each component of the measured field varies, in the process of measuring, by not more than 2% of the field magnitude (a condition that is usually met by variations of the Earth's magnetic field). Absoluteness of the measurements implies that the shift of the measured parameter (the value of the field component) produced by the measuring process does not exceed the inaccuracy of the sensor in measuring variations of the magnetic field modulus. This, evi-

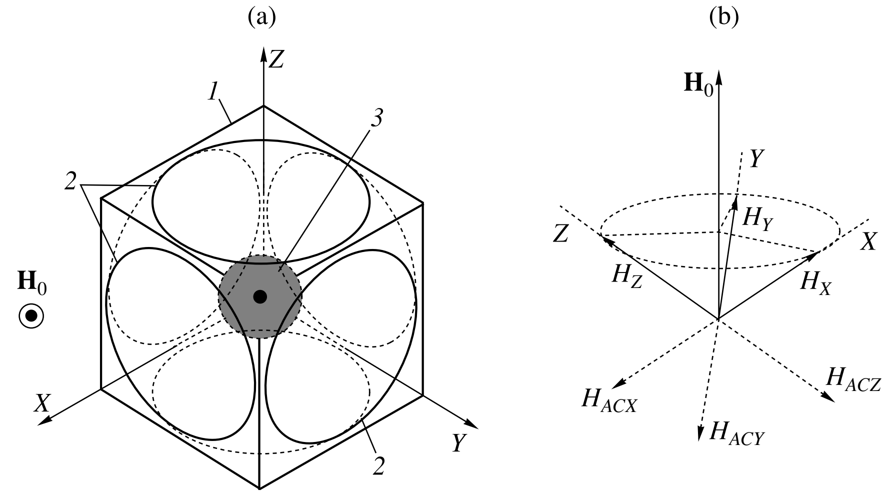

Fig. 1. (a) The sensor in a symmetric three-component system of coils. (*1*) Cubic frame of the coil system; (*2*) coils; (*3*) sensor. The axis of the sensor and the vector of the measured field $\mathbf{H}_0$ are directed along the normal to the plane of the figure. (b) The projections H_X, H_Y, and H_Z of the measured magnetic field $\mathbf{H}$ and the ac compensating fields in the coils H_{ACX}, H_{ACY}, and H_{ACZ} (maximum values). The axis of the sensor and the vector of the measured field lie in the plane of the figure. The circumference in the *XYZ* plane is the hodograph of the total magnetic field vector.

dently, does not solve the problem of the absolute accuracy of the sensor's readings. This question is, however, well studied as applied to the sensors of quantum magnetometers, and the absolute error of the best ones is known to lie in the range 0.1–0.01 nT, being practically independent of the field magnitude to be measured [10].

As the sensor, it is proposed to use a cesium or potassium optically pumped magnetometer tracking the oscillating signal of transverse magnetization of the atoms in the cell (a so-called M_x magnetometer, characterized by a high accuracy and high speed).

The gist of the method is to create, at the point of the sensor, a system of compensating fields varying harmonically in such a way that the vector of the total magnetic field at the sensor rotates around the initial direction of the magnetic field, keeping a constant magnitude and passing, on each round, through three points at each of which two components of the magnetic field are compensated with high precision and the third component is uncompensated and amenable to measuring.

The proposed method is suitable for a wide range of magnetic fields but is most efficient for precision measurements of components of the Earth's magnetic field because of its uniformity and the relative smallness of its variations.

DESCRIPTION OF THE METHOD

The sensor is placed in the center of a symmetric three-component system of magnetic coils. The system is oriented so that both the principal diagonal of a cube inscribed into the system of coils and the symmetry axis of the sensor (coincident with the direction of propagation of the pump) are directed along the Earth's magnetic field $\mathbf{H}_0$ (Fig. 1a). The chosen coordinate system is rigidly bound to the axes of the coil system. In this system, all three components of the Earth's magnetic field vector are initially equal in magnitude: $H_X = H_Y = H_Z = |\mathbf{H}_0|/\sqrt{3} = H_0$.

The field H_{ACi} in each coil ($i = X, Y, Z$; the subscripts *AC* indicate oscillating values) is initially chosen to completely compensate the corresponding component of the Earth's magnetic field H_i (Fig. 1b). In practice, the initial choice of the fields and the orientation of the coil system are carried out using a priori information about the measured field vector. After that, all the components are automatically compensated using a system of feedback loops.

When the compensating fields are turned on simultaneously in all three coils, the total field at the sensor vanishes. Switching the field of the ith coil off ($i = X, Y, Z$) leads to the appearance of the corresponding uncompensated ith component, which can be measured by the sensor. The accuracy of the measurement, in this case, is higher by a few orders of magnitude than the accuracy of compensation of the orthogonal components of the field because the contribution of the uncompensated orthogonal components is proportional to $(1 - \cos\alpha)$, where α is a small angle (Fig. 2).

A cycle of such measurements over $i = X, Y$, and Z will provide full information about all three components of the field. This information, in turn, is used in real time to refine the values of the compensating fields H_{ACX}, H_{ACY}, and H_{ACZ} in the coils X, Y, and Z, thereby providing three feedback loops.

The next step in the development of the proposed approach consists in passing from discrete changes of the field to continuous or quasi-continuous measurements: we make the field rotate continuously or quasi-continuously (discretely, with a small step) in such a way that three points on the circumference of rotation meet the above conditions $H_{ACi} = 0$ ($i = X, Y, Z$). The total magnetic field vector, in this case, is always tilted by 35.2° with respect to the axis of optical pumping of the M_x magnetometer. Thus, continuity of resonance locking is provided, and real measurements are made at three points of the circumference. It is important that the measurement of the component H_i is performed, as before, at the moment when the corresponding compensating field H_{ACi} is turned off and only the compensating fields orthogonal to H_i are on. In this way, absolute measurements of the field components are realized.

To provide a high accuracy of compensation of the orthogonal components of the field, it is proposed to use the procedure of self-calibrating by minimizing the field modulus under variation of the compensating fields.

As a working medium of the sensor, potassium may be used (its drawback is a large size of the cell and, hence, of the whole system), or cesium with a small-size cell and a relatively broad asymmetric resonance line (the drawback of cesium is a reduced accuracy). Note that a possible asymmetry of the resonance line will lead to the same error for all the components and practically will not affect the accuracy of measurement of the magnetic field direction.

By the example of measuring the X component of the field, let us estimate the requirements on the accuracy of transverse component compensation. Hereinafter, we assume that the modulus of the measured field equals 50 000 nT, the mean value of any field component is $|\mathbf{H}_i| = |\mathbf{H}_0|/\sqrt{3} \approx 28867$ nT, the maximum variation of the field component is ±1000 nT, and the required absolute accuracy of the device is ±0.1 nT.

Thus, the error introduced into the measurement of the X component due to inaccuracy in compensation of the Z component (Fig. 3) should not exceed ΔH_X = 0.1 nT. Therefore, the maximum value of the field in the Z component is $\Delta H_Z = H_i \sin[\arccos(1 - \Delta H_X/H_X)]$ = 76 nT; in the presence of two components that are simultaneously not completely compensated, $\Delta H_Y = \Delta H_Z$ = 54 nT. In other words, the relative error of transverse component compensation may reach 1.8×10^{-3}. This means that the coil system may be manufactured using any material that provides constancy of the geometrical shape of the system (orthogonality of the coils) with no special requirements on the size stability of the coil system.

Before proceeding to an analytical description of the signal in the system and to the results of numerical simulation, let us enumerate the main drawbacks of this method and possible ways of their elimination.

(i) The requirements for orthogonality of the coils are rather stringent; namely, the limitation of the error by a value of 0.1 nT leads to a requirement on orthogonality of the coil axes at a level of 3×10^{-6} (3 μm per meter, or 0.6 seconds of arc), which can hardly be achieved. Therefore, the nonorthogonality of the coils in the system should be measured after manufacturing (or should be measured periodically in the process of calibration) and should be compensated electronically. Then the estimate of 0.6″ will refer to the variation of the angles between the coils of the system. The procedure of calibration of systems characterized by an imperfect orthogonality of the axes is described in [11].

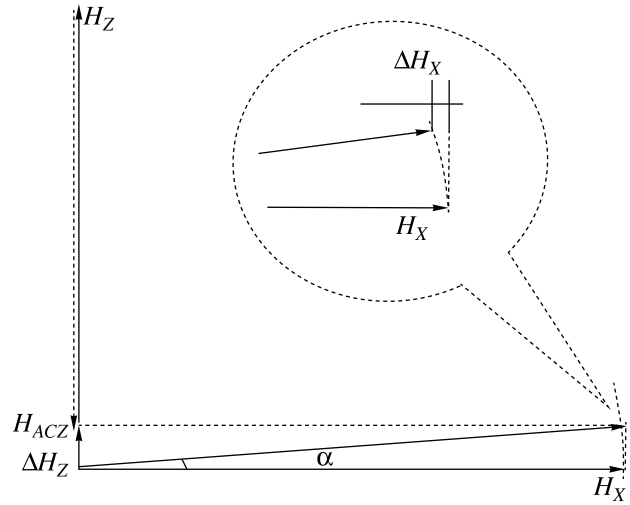

Fig. 2. The error of measuring the H_X component of the field related to inaccuracy in compensating the H_Z component. ΔH_Z is the residual field along the Z axis and ΔH_X is the error of measuring the component H_X.

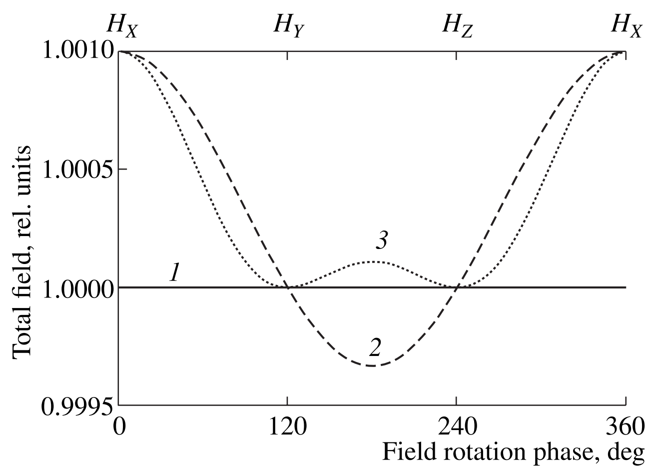

Fig. 3. The dependence of the modulus of the total magnetic field vector on the rotation angle (the plot is normalized to H_0): (*1*) for the direction of the measured field coincident with the axis of the system ($\Delta H_i = 0$, $\Delta H_{Mi} = 0$), (*2*) for the variation of the X component of the measured magnetic field ($\Delta H_X = 1.001$, $\Delta H_{MX} = 0$), and (*3*) for the variation worked out by the feedback system ($\Delta H_X = 1.001$, $\Delta H_{MX} = 1.001$).

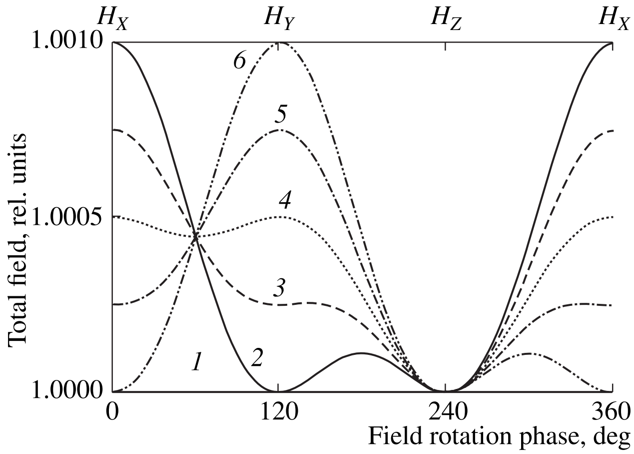

Fig. 4. The response of the system to different variations of the X and Y components of the measured magnetic field (the plot is normalized to H_0): (*1*) no deviation of the vector, (*2*) variation of the X component of the field, (*3–5*) intermediate cases, and (*6*) variation of the Y component of the field.

(ii) The inertia of the magnetic coils and of the sensor will lead to a delay in the measurement of the magnetic field and, as a consequence, to a shift of the points on the circumference at which the measurements are made. The negative effect of this delay is rather small because, as will be shown below, the derivative of the field modulus with respect to the field rotation phase, in steady-state conditions, vanishes at these three points (Fig. 4). When the system is required to be fast, the most efficient way to suppress the effect of this delay is to create conditions where the direction of the magnetic field constantly coincides with the principal axis of the system (see (iii) below).

(iii) The constancy of the modulus of the magnetic field upon its rotation holds only for small angular deviations of the measured magnetic field from the axis of the coil system. For large angular deviations of the field, the rate of variation of the field modulus upon its rotation increases (Fig. 3), which may reduce the accuracy of the measurements due to a sluggishness of the system and even disable the sensor. This drawback may be eliminated in different ways:

(a) By continuously compensating the field variations using slow (compared with the rotation period) compensating fields H_{DCi} of the coils. In this case, the value of the measured field component is determined both by the readings of the sensor and by the current in the coil; thus, strictly speaking, the method loses its absoluteness. However, when the variations of the Earth's magnetic field do not exceed 2% (or 1000 nT), the introduction of such compensating fields leads to a requirement for stability of the coil system parameters at a level of 10^{-4}, which may be easily implemented in practice.

(b) By using a mechanical servo system. This method does not affect the absoluteness of the measurements but is much more complicated because the required accuracy of field measurement of about 0.1 nT against a background of ~50000 nT implies an accuracy of mechanical positioning better than 0.4 arc seconds. Still, technically, it is realistic; such accuracies are achieved in the drives of astronomical telescopes.

In both cases, additional "slow" feedback systems ensure constancy of the total magnetic field modulus at all points of the circumference. Closely lying points of the circumference become equivalent; this fact sharply decreases the requirements on the accuracy of choice of the points on the circumference at which the magnetic field is measured and makes it possible to pass from measuring the magnetic field at three points on the circumference to measuring it on three segments of the circumference (see below).

(iv) The proposed method is characterized by a reduced signal-to-noise ratio and, hence, by a lower short-term sensitivity. This is related to the fact that the measurement of each component occurs only at a single point of the field rotation circumference. Therefore, in the case of quasi-continuous rotation of the field over a circumference consisting of N points, the sensitivity in each component decreases, as compared with the sensitivity of the modulus sensor, by a factor of $N^{1/2}$ (the sensitivity of the sensor is assumed to be limited by the Gaussian shot noise of the photocurrent). In what follows, however, it will be shown that the effective range of measurements may be substantially widened without loss of absoluteness.

(v) The rotation of the magnetic field at a frequency of ω around the axis of the M_x sensor is known [1] to cause a shift of the measured field strength by $\pm\omega/\gamma$ (where γ is the gyromagnetic ratio) depending on the rotation direction (the gyroscopic effect). However, since the rotation frequency is known, this effect can be easily taken into account without a loss in accuracy of the measurements.

DESCRIPTION OF THE SIGNAL IN THE SYSTEM

Consider the signal in the system in the absence of additional systems of field variation compensation. The law of the time variation of the compensating fields H_{ACi} should satisfy the following conditions:

1. Constancy of the total field modulus under the condition that the measured field is parallel to the principal axis of the system.

2. Constancy of the angular velocity of rotation of the total field vector.

3. The presence of three points on the circumference at the angles φ_i^0 (where $\varphi_X^0 = 0°$, $\varphi_Y^0 = 120°$, and $\varphi_Z^0 = 240°$ and the angles are measured in the XYZ plane perpendicular to the principal axis of the system from the projection of the X axis onto this plane), with one component at each of these points being equal to zero and the other two equal in magnitude to the corresponding

components H_{ACi} and opposite in sign. For instance, at the point $\varphi = \varphi_X^0 = 0°$, $H_{ACX} = 0$, $H_{ACY} = -H_Y$, and $H_{ACZ} = -H_Z$.

One can easily show that all these conditions are satisfied by the expression

$$H_{ACi}(t) = (2/3)[\cos\varphi_i(t) - 1]H_{Mi}, \quad (1)$$

where H_{Mi} is the measured value of the ith component of the field (ideally, $H_{Mi} = H_i$),

$$\varphi_X(t) = \omega t, \quad \varphi_Y(t) = \varphi_X(t) - \varphi_Y^0,$$
$$\varphi_Z(t) = \varphi_X(t) - \varphi_Z^0.$$

If the compensating fields vary according to Eq. (1), we can calculate the response of the system to a variation ΔH_i (hereinafter, $\Delta H_i = H_i - H_0$ and $\Delta H_{Mi} = H_{Mi} - H_0$) of any component of the measured magnetic field (Fig. 3, curves *1*, *2*). The time dependence of the response signal is close to sinusoidal (a signal is understood hereinafter as an instantaneous value of the field modulus measured by the sensor), and at three points $\varphi_i^0 = 0°$, 120°, and 240°, the total field modulus reproduces (not yet perfectly) the values of the three components of the measured field:

$$H(t) = \sqrt{\sum_i (H_i + H_{ACi})^2}$$
$$\approx H_0 + \frac{1}{3}\sum_i \Delta H_i[1 + 2\cos\varphi_i(t)]. \quad (2)$$

The results of measurements of these three components should be used by the system to correct the values of H_{ACi} so that $H_{Mi} = H_i$. Curve *3* in Fig. 3 shows the result of operation of the feedback system. Now, the modulus of the total magnetic field at the points where the measurement is performed exactly reproduces the values of the three components of the measured field, while the derivative of the total modulus with respect to phase (and, therefore, with respect to time) at these points vanishes:

$$H(t) \approx H_0 + \frac{1}{9}\sum_i \Delta H_i[1 + 2\cos\varphi_i(t)]^2. \quad (3)$$

Figure 4 shows the responses of the system to variations of the measured field vector. One can see that, in all cases, the derivative remains zero at the points where the measurement is performed. Now we may formulate the requirements on the accuracy of choice of these points and (on replacing a point by a segment on the circumference) on the width of a segment providing an accuracy of 0.1 nT. These requirements are most stringent for the greatest field variations $\Delta H_i = 1000$ nT: $\Delta\varphi = \pm 0.5°$. In this case, the sensitivity of measuring a field component on a segment 1° in width appears to be lower than the sensitivity of a scalar sensor by a factor of $\sqrt{360} \approx 19$.

With decreasing variations of the measured field, the width of the segment increases: for $\Delta H_i = 50$ nT, $\Delta\varphi = \pm 3°$, for $\Delta H_i = 5$ nT, $\Delta\varphi = \pm 10°$, and for $\Delta H_i = 0.5$ nT, $\Delta\varphi = \pm 33°$. Thus, the introduction of additional systems of compensation of the field variations will allow one to widen the time range of useful measurements over the whole circumference.

In addition, even with no system of compensation of the field variations, the signal can be processed in real time, which also makes it possible to widen the time range of useful measurements over the whole circumference. Consider the most general case where the feedback system is operative (and, hence, $\Delta H_{Mi} \approx \Delta H_i$) but, at the same time, there also exist fast fluctuations of the measured field Δ_i not suppressed completely by the feedback system, so that $\Delta H_i = \Delta H_{Mi} + \Delta_i$. In turn, the variations of the coefficients k_i of the coils contribute to the value of ΔH_{Mi}:

$$\Delta H_{Mi} = (H_0 + \Delta H'_{Mi})(1 + \Delta k_i) - H_0$$
$$\approx \Delta H'_{Mi} + \Delta k_i H_0. \quad (4)$$

Here, $\Delta H'_{Mi}$ is the result of the measurement proper and $\Delta k_i = k_i - 1$ are the deviations of the transfer coefficients of the coils k_i from unity; the values Δk_i take into account both variations of the geometrical factor of the coils and drifts of the current source.

Then,

$$H(t) \approx H_0 + \frac{1}{9}\sum_i \Delta H_{Mi}[1 + 2\cos\varphi_i(t)]^2$$
$$+ \frac{1}{3}\sum_i \Delta_i[1 + 2\cos\varphi_i(t)]. \quad (5)$$

This signal can be represented in the form

$$H(t) \approx H_0 + A + B\cos\omega t + C\sin\omega t$$
$$+ D\cos 2\omega t + E\sin 2\omega t, \quad (6)$$

where

$$A = \frac{1}{3}\sum_i \Delta H_i,$$
$$B = \frac{2}{9}\sum_i (3\Delta H_i - \Delta H_{Mi})\cos\varphi_i^0,$$
$$C = \frac{2}{9}\sum_i (3\Delta H_i - \Delta H_{Mi})\sin\varphi_i^0, \quad (7)$$
$$D = \frac{2}{9}\sum_i \Delta H_{Mi}\cos 2\varphi_i^0,$$
$$E = \frac{2}{9}\sum_i \Delta H_{Mi}\sin 2\varphi_i^0.$$

Parameter A in Eq. (6) can be measured in real time by averaging the signal over the period of modulation (rotation of the field) T_{mod}; parameters B and C can be measured by lock-in detection at the frequency ω; and parameters D and E, by lock-in detection at the frequency 2ω:

$$A = \frac{1}{T_{mod}}\int_0^{T_{mod}} S(t)dt, \quad B = \frac{2}{T_{mod}}\int_0^{T_{mod}} S(t)\cos\omega t dt,$$
$$C = \frac{2}{T_{mod}}\int_0^{T_{mod}} S(t)\sin\omega t dt,$$
$$D = \frac{2}{T_{mod}}\int_0^{T_{mod}} S(t)\cos 2\omega t dt, \tag{8}$$
$$E = \frac{2}{T_{mod}}\int_0^{T_{mod}} S(t)\sin 2\omega t dt.$$

Then,

$$\Delta H_X = A + (B + D),$$
$$\Delta H_Y = A - (B + D)/2 + \sqrt{3}(C - E)/2, \tag{9}$$
$$\Delta H_Z = A - (B + D)/2 - \sqrt{3}(C - E)/2.$$

As a result, the values of ΔH_i will be calculated using all the information over the period, with the signal-to-noise ratio reduced by only a factor of 2 (due to the high efficiency of lock-in amplification) compared with that of the modulus sensor; the sensitivity of measuring a field component is lower by a factor of $\sqrt{2}$ than that of measuring the field modulus and is higher by a factor of $(N/2)^{1/2}$ than the sensitivity of measuring each component at one point per modulation period.

Then, it follows from Eqs. (4) and (7) that the lock-in detection of the signal at the frequency 2ω allows one to obtain independent information about the coefficients k_i of the coils, or, more exactly, about the deviation of Δk_i from their mean values (at a given moment). The accuracy of this measurement is of the same order as the accuracy of measurement of the field components. In particular, when systems with slow compensation of the measured field variations are used,

$$\Delta k_X - \frac{1}{3}\sum_i \Delta k_i = \frac{1}{H_0}3D,$$
$$\Delta k_Y - \frac{1}{3}\sum_i \Delta k_i = \frac{1}{H_0}\left(-\frac{3\sqrt{3}}{2}E - \frac{3}{2}D\right), \tag{10}$$
$$\Delta k_Z - \frac{1}{3}\sum_i \Delta k_i = \frac{1}{H_0}\left(\frac{3\sqrt{3}}{2}E - \frac{3}{2}D\right).$$

According to Eq. (10), the coefficients of the coils can be equalized in the process of field measurement. This procedure does not improve the accuracy of measurement of the field modulus but makes it possible to eliminate the effect of coil drift on the accuracy of measuring the field direction.

RESULTS OF NUMERICAL SIMULATION

To numerically simulate the process of measuring the magnetic field, a program was created, whose operation capacity was tested by simulating a scalar M_x magnetometer with known parameters. The kernel of the program is a model of the M_x magnetometer represented as a system of phase locking (PL) capable of searching for the magnetic M_x resonance and locking to it. The M_x resonance can be described by a steady-state solution of the Bloch equation [1]. The model takes into account the shot noise of the sensor photocurrent and the phase delay of the signal in the PL loop. The virtual M_x magnetometer is placed into a magnetic field varying according to expressions (1) and (2); the signal of the time variation of the field modulus is used as an input for the feedback systems.

The program allows one to vary the parameters of the magnetic field (the magnitudes of the components, drift velocity, and values of jumps), the parameters of the M_x resonance (the amplitude, linewidth, field-induced broadening, phase shift, and noise density), the parameters of the device (the signal digitization rate, gain factors, thresholds, time delays, etc.), and the defects of the coil system (the variations of the coefficients and the nonorthogonality of the axes).

The program simulates the behavior of the system (Fig. 5) and yields the mean error of the measurements, variance of the measurements, and variance of the measurement error for a chosen accumulation time.

Two models were studied: (1) with measurements at three points of the circumference and (2) with lock-in detection of the signal in accordance with Eqs. (5)–(8). Both models were examined in a stable field: (A) with no system of compensation of the field variations and (B) with systems of compensation. It was assumed that the compensation of the field variations and the field modulation are performed using the same coils. The model based on the mechanical servo system was not studied separately because it is equivalent to the model with field variation compensation in the case of zero deviation of the direction of the field **H** from its initial value. The nonorthogonality of the coils was taken into account by introducing the constants k_{ij} describing the contribution of the ith coil to the field H_j.

Systems with a potassium sensor with the resonance linewidth Γ = 1 nT were simulated, as well as systems with cesium censors with Γ = 20 nT characterized by the intrinsic short-term sensitivity $\sigma_{0.1s}$ = 10 pT. As expected, the use of the sensor with a broad resonance line, at the expense of reduced sensitivity, provided, in

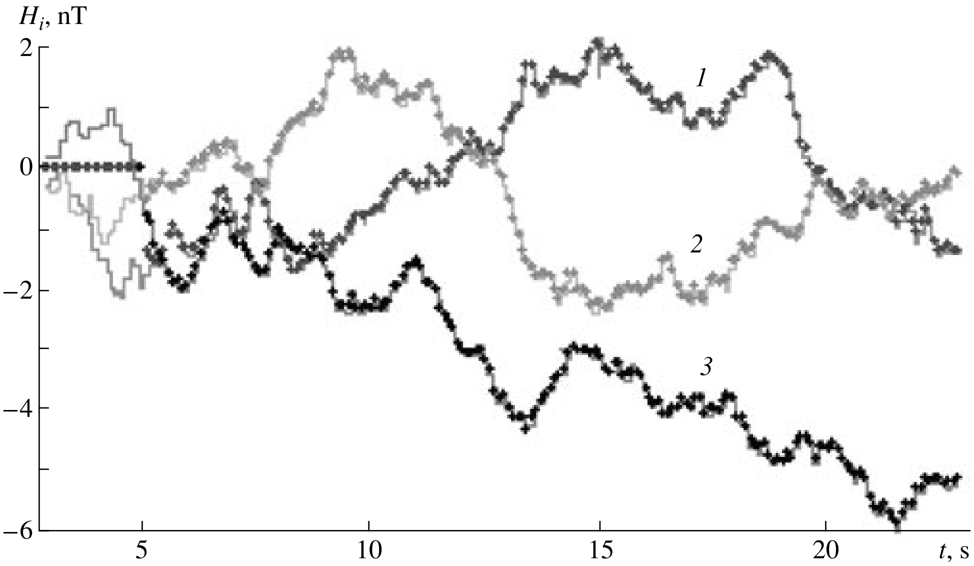

Fig. 5. Numerical simulation of the phase-locking process and measurements of the three components of the drifting magnetic field. The solid lines are the components of the measured field; the crosses, measured values of the field. (*1*) H_X, (*2*) H_Y, and (*3*) H_Z.

this scheme, advantages both in the width of the frequency locking range and in the field tracking speed. Below, the results obtained for this particular model are presented. The field was rotated in a quasi-continuous way with the sampling rate F_S = 10 kHz. Accordingly, the period of digitization of the signal and its processing by the PL circuit of the modulus sensor was chosen to be $T_S = 1/F_S$ = 0.1 ms with the effective signal time delay $T_d = T_S/2$. The gyroscopic effect was not taken into consideration. Initial averaging of the results was performed with a sampling period of T_M = 0.1 s. To reduce statistical error, data at each point were accumulated up to 10^4 readings. A procedure of equalization of the coil coefficients k_i according to (10) was not included in the algorithms of the basic models but was tested separately.

Figure 6 shows the results of studying the short-term (0.1 s) sensitivity σ_X in the X component and the errors in the components Δ_{max} = max($|\Delta_X|$, $|\Delta_Y|$, and $|\Delta_Z|$ as functions of the modulation period T_{mod} (and, correspondingly, of the number of points per period $N = T_{mod}/T_S$) at ΔH_X = 10 nT. One can see that, for T_{mod} > 3 ms (N > 30), model 2B provides the smallest errors (at the level of noise on averaging over 10^4 measurements) with the highest sensitivity. In this case, an effective time constant of the response $\tau \le 0.1$ s is provided at $T_{mod} \le 50$ ms. The parameters of model 2A approach those of model 2B at large T_{mod}, but model 2A, in this case, requires matching the amplification coefficients in the feedback loops with an accuracy of 1%, whereas model 2B does not have this drawback. For small T_{mod}, the error does not depend on the fashion of signal processing and is determined by the values of the field deviation and the signal time delay.

Figure 6a confirms the conclusion of the preceding section about the dependence of the sensitivity of measuring the field component on the number of points in a modulation period.

The numerical simulation showed that the errors of the models with no field variation compensation (1A and 2A) are proportional to the deviation of the measured field ($\Delta_i/\Delta H_i \approx 0.001$). This drawback is absent in

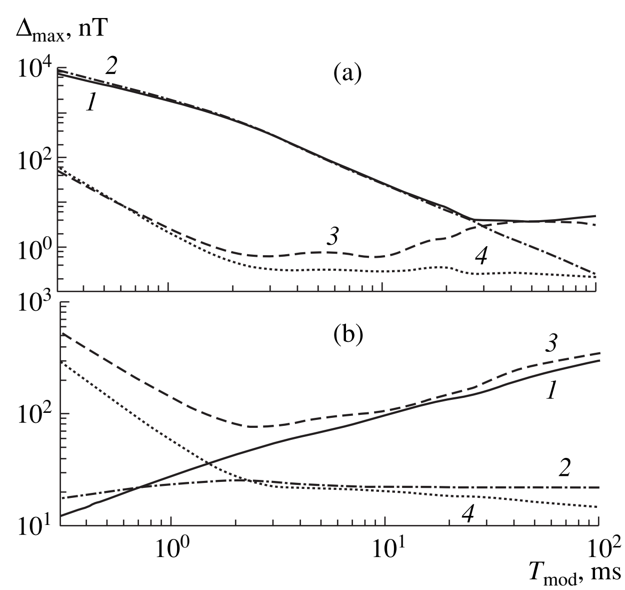

Fig. 6. (a) The dependence of the error in the components Δ_{max} = max(Δ_X, Δ_Y, Δ_Z) for ΔH_X = 10 nT and $\Delta H_Y = \Delta H_Z = 0$ on the modulation period T_{mod}; (b) the dependence of the short-term (0.1 s) sensitivity σ_X in the X component on the modulation period T_{mod} (*1*) 1A, (*2*) 2A, (*3*) 1B, and (*4*) 2B.

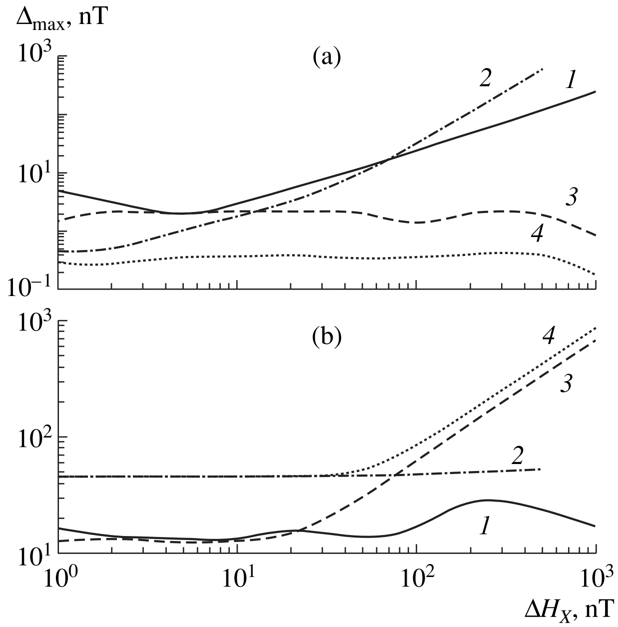

Fig. 7. (a) The dependence of the error in the components $\Delta_{max} = \max(\Delta_X, \Delta_Y, \Delta_Z)$ on the variation of the field component ΔH_X for $\Delta k_i = 0$; (b) the dependence of the error increment in the components on the variation of the field component ΔH_X in the case of variation of the coefficient k_X of the X coils by the value $\Delta k_X = 0.001$ for $\Delta k_Y = \Delta k_Z = 0$. (*1*) 1A, (*2*) 2A, (*3*) 1B, and (*4*) 2B.

the models with systems of field variation compensation (1B and 2B). However, variations of the coefficient of the compensating coil $\Delta_i/\Delta H_i = \Delta k_i$ contribute directly to the results of measurements in these models. In the absence of variations of the coefficient of the compensating coil, the smallest shifts (below 0.05 pT, i.e., at the level of scatter of points upon averaging) are exhibited by model 2B, with lock-in detection over the whole modulation period and with field variation compensation.

For the width of the sensor resonance line $\Gamma = 20$ nT, short-term sensor sensitivity $\sigma_{0.1s} = 10$ pT, modulation period $T_{mod} = 50$ ms, and amplification factor in the feedback loop of the sensor $k_{FB} = 10^4$, model 2B showed the following parameter values:

—response time (at the level of 0.7) $\tau = 0.1$ s;

—short-term sensitivity for the time constant 0.1 s in the field components $\sigma_i = 14.8$ pT;

—range of initial locking $|\Delta H_{i0\max}| = 730$ nT;

—range of tracking of the field variations $|\Delta H_{i\max}| \gg 1000$ nT;

—greatest variation of the angle between orthogonal coils providing an absolute accuracy of ±0.1 nT $|\Delta\beta| = 0.6''$;

—greatest drift of the magnetic coil system providing an absolute accuracy of ±0.1 nT $|\Delta k_{i\max}| = 115 \times 10^{-6}$.

In using a mechanical servo system, the admissible drift of the magnetic coil system may reach $\pm 1350 \times 10^{-6}$ (neglecting intrinsic errors of the servo system).

CONCLUSIONS

A method of simultaneous measurement of the three components of the Earth's magnetic field characterized by a high absolute accuracy with a short measuring time was proposed. An analytical calculation and numerical simulation of a measuring procedure based on the proposed method was presented. It was shown that, using an optically pumped M_x magnetometer and a three-component symmetric system of magnetic coils, it is possible to simultaneously measure, during 0.1 s, all three components of the Earth's magnetic field with an absolute accuracy of ±0.1 nT.

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