ISSN 0030-400X, Optics and Spectroscopy, 2006, Vol. 100, No. 1, pp. 12–14. © Pleiades Publishing, Inc., 2006.
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# Phase Error Elimination in the $M_x$ Magnetometer and Resonance Line Shape Control in an Unstable Field Using the Technique of Invariant Mapping of a Spin Precession Signal

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Received May 24, 2005

**Abstract**—A method of two-dimensional mapping of a magnetic resonance signal is proposed that allows one to exclude the effects of field variations on the processes of monitoring of the magnetic resonance spectra and phase correction in the feedback loop of the $M_x$ magnetometer.

PACS numbers: 32.60

**DOI:** 10.1134/S0030400X06010048

All the devices of quantum magnetometry are based on measuring a magnetic resonance frequency. In proton magnetometers, this is the frequency of free precession of a proton spin; in optically pumped magnetometers, this is the frequency of magnetic resonance of the total angular momentum of the atom [1]. With respect to the magnetic resonance detection technique, quantum magnetometers may be subdivided based on several criteria. We will be interested in the division of magnetometers into (i) devices with a signal proportional to the longitudinal component of the magnetic moment (an $M_z$ signal) and (ii) devices tracing the phase of the oscillating transverse component of the magnetic moment. The devices of the first type ($M_z$ magnetometers) are relatively slow but more accurate, while those of the second type are faster ($M_x$ magnetometers). The errors in both types of devices are associated both with parametric shifts of the magnetic resonance proper and with the errors in determination of the resonance line center. The latter errors are typical of the $M_x$ magnetometers, which need the phase shift of the magnetic moment precession signal to be set with high accuracy in the feedback loop of the device.

By the $M_x$ signal, we will mean below the result of lock-in amplification HYPERLINK of the primary spin precession signal at the frequency ω of the ac field $H_1$. The $M_x$ signal is characterized by the amplitude of the oscillations, by their phase with respect to that of the ac magnetic field inducing the resonance, and by the width of the resonance, which depends both on the relaxation processes and on the degree of saturation of the resonance. Determination of the phase of the magnetic resonance for $M_x$ magnetometers is of crucial importance because inaccuracy in the phase shift compensation leads to an error in determination of the resonance center and, as a consequence, to a systematic error in the magnetic field measurement. The most straightforward way to correct the phase is to symmetrize the magnetic resonance line, i.e., the dependence of the $M_x$ signal on the frequency ω of the field $H_1$. However, the time needed to record the magnetic resonance line is much larger than the inverse resonance width, and, under conditions of a real (i.e., unstable) magnetic field, the accuracy of such a procedure is limited by the field variations. In some cases, such a measurement turns out to be completely impossible.

In what follows, we propose a method for representation of the magnetic resonance signal that allows one, under conditions of significant random variations of the magnetic field, to perform reliable phase correction, to measure the amplitude and saturation factor of the $M_x$ resonance, and to identify the presence of neighboring spectral components in the magnetic spectrum.

The $M_x$ signal $S$, related to the transition between the two magnetic sublevels in the magnetic field $H_0$ in the presence of an ac radio-frequency field $H_1$ at the frequency ω, is described by the well-known Bloch equations [2], whose steady-state solution in a complex form has the form

$$S = \frac{A\omega_1}{Z^2}\frac{i + \delta Z}{1 + \delta^2}e^{i\varphi}, \tag{1}$$

where $\delta = (\omega - \omega_0)/\Gamma Z$ is the reduced detuning of the radio frequency ω from the resonance frequency $\omega_0$, φ is the additional increment of the phase related both to the phase retardation in the measurement channel and to the experimental geometry, $A$ is the amplitude of the resonance signal, $\Gamma = 1/T_2$ is the resonance width, $Z = 1 + \omega_1^2 T_1 T_2$ is the saturation factor of the resonance,

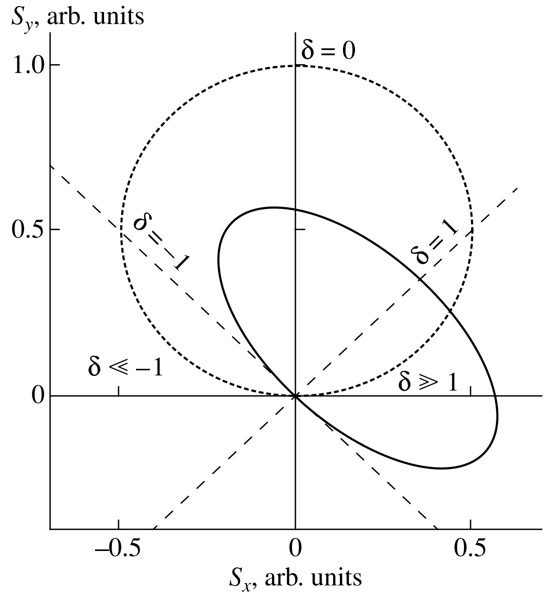


**Fig. 1.** $M_x$ resonance signal on the complex plane. The solid line is the diagram in the laboratory coordinate system *xy*; the dotted line, the diagram in the natural coordinate system *x'y'*.

$\omega_1$ is the Rabi nutation frequency, and $T_1$ and $T_2$ are the longitudinal and transverse relaxation times of the magnetic moment.

At $\varphi = 0$, the components $S_x = \mathrm{Re}S$ and $S_y = \mathrm{Im}S$ are represented by a Lorentzian absorption profile peaked at zero detuning and by a Lorentzian dispersion profile with zero value at zero detuning. It is the latter component of the signal that is used in systems of autotuning of the $H_1$ field frequency to the resonance value [3].

Let us represent the signal $S = S_x + iS_y$ on a complex plane. In the general case, it has the shape of an ellipse with the ratio of axes equal to the saturation factor $Z$ and with the minor axis going from the coordinate origin and tilted with respect to the $y$ axis at the angle $\varphi$. The ellipse represents a locus of the $M_x$ signal for all detunings $\delta$, which makes this representation attractive. A remarkable feature of this representation is its invariance with respect to magnetic field fluctuations: under random variations of the detuning caused by these fluctuations, the current point randomly moves over the contour of the ellipse, whose shape and orientation remain stable at fixed values of the amplitude $A$, phase $\varphi$, and saturation factor $Z$. This circumstance allows one to reliably measure these three values in spite of the external field fluctuations.

Thus, optimization of the regime of the $M_x$ magnetometer is reduced to choosing the phase $\varphi$ so that the ellipse representing a multitude of the signals $S$ has the minor axis oriented along the $y$ axis. The ratio of the axes depends on the amplitude of the magnetic field $H_1$ and should have a value close to 2, which corresponds to the greatest resolving power of the resonance.

A simple method of numerical treatment of such a diagram consists of going to a new coordinate system $x'y'$ (further referred to as "natural"), in which the ellipse is transformed into a circle with the center lying on the $y'$ axis (Fig. 1):

$$|S'| = (A\omega_1/Z)\sin\psi, \quad (2)$$

where $\psi = \mathrm{arccot}\,\delta$ is the angle on the complex plane.

Such a transition is carried out by choosing the coefficients $\varphi$ and $Z$ upon rotation of the coordinate system $xy$ by the angle $\varphi$ with subsequent scaling of the $y$ axis by a factor of $Z$:

$$\begin{aligned} x' &= x\cos\varphi + y\sin\varphi, \\ y' &= Z(-x\sin\varphi + y\cos\varphi). \end{aligned} \quad (3)$$

Comparison of the diagram in the new graphic representation with an ideal circle allows one to easily

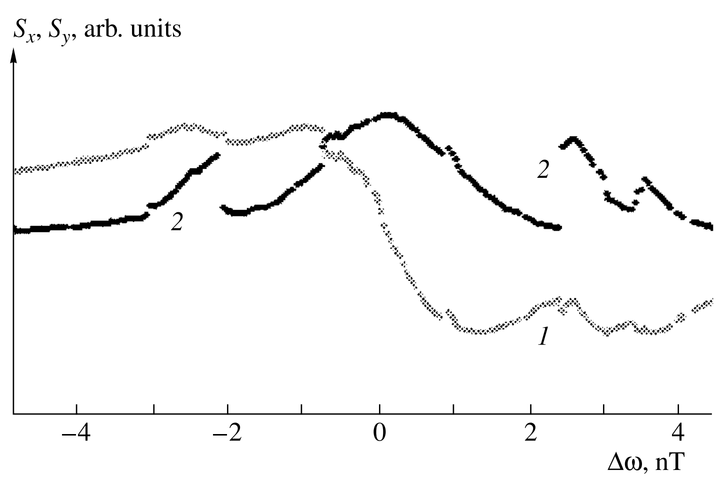


**Fig. 2.** Record of the $M_x$ resonance components in a real magnetic field. Scanning rate 0.3 nT/s. (*1*) $S_x$; (*2*) $S_y$.

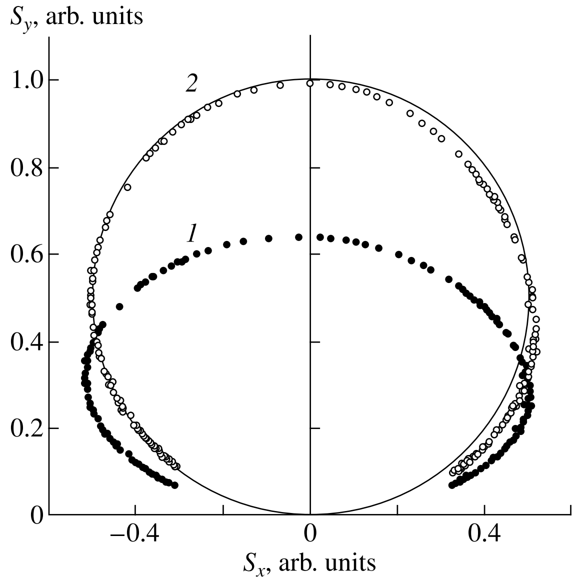


**Fig. 3.** The same record of the $M_x$ resonance components in a real magnetic field as in Fig. 2 (*1*) in the laboratory coordinate system and (*2*) in the natural coordinate system.

detect distortions in the line profile related to the presence of additional spectral components in the magnetic spectrum.

Practical implementation of two-dimensional mapping of the signal is performed by simultaneous lock-in amplification of the original $M_x$ signal of precession using two lock-in amplifiers, with the phases of their reference signals shifted by 0° and 90° with respect to that of the radio-frequency field. The outputs of the amplifiers are displayed, respectively, along the *x* and *y* axes of a monitor.

Figure 2 gives an example of a conventional record of a narrow magnetic resonance in the spectrum of $^{41}K$ (the $F = 2, m_F = 1 \longleftrightarrow F = 2, m_F = 2$ transition) in a real field. One can see that the dependences $S_x(\delta)$ and $S_y(\delta)$, due to the field fluctuations, strongly differ from ideal Lorentzian dispersion and absorption curves.

Figure 3 shows (*1*) the same record represented on the complex plane and (*2*) the result of transformation of the laboratory coordinate system *xy* to the natural system *x'y'* with the transformation parameters $Z = 1.55 \pm 0.02$ and $\varphi = -1.5° \pm 0.5°$ (for an intrinsic width of the resonance $\Gamma = 1$ nT, the phase error of 0.5° corresponds to an error in position of the resonance center of 9 pT). The distortions of the shape of the signal on the right-hand side of the diagram indicate the presence of additional components in the magnetic spectrum (related, as a rule, to the ac component of the field at a frequency of 50 Hz). By expanding the scanning range, it would be possible to detect the distortions related to the presence of the neighboring line $F = 2, m_F = 0 \longleftrightarrow F = 2, m_F = 1$.

Thus, the advantage of the proposed method of representation of the magnetic resonance signal over the conventional one is not only that the rotation of the diagram around the origin of coordinates, the eccentricity of the ellipse, and the distortion of its shape can be easily noticed and measured with a high accuracy (Fig. 3) but also, first of all, that such a measurement does not require the magnetic field to be stabilized and may be realized in nonlaboratory (field) conditions, i.e., under usual conditions of operation of quantum magnetometers.

Indeed, no matter how large the magnetic field fluctuations during scanning of the radio frequency through the magnetic resonance, they will change only the density of measurements on the diagram and not its shape. By repeating the scanning procedure, one can accumulate a sufficient number of points to determine the phase shift and other parameters of the resonance with any accuracy, whereas recording the resonance line in a drifting field with the use of conventional methods of accumulation does not improve the accuracy of measurements.

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*Translated by V. Zapasskiĭ*