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EXPERIMENTAL INSTRUMENTS AND TECHNIQUES

Optically Pumped Quantum M_X Magnetometers: Digital Measurement of the M_X Resonance Frequency in a Rapidly Varying Field

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Abstract—A new way of digitally constructing a phase-locked loop for an optically pumped quantum M_X magnetometer is suggested. It provides a high accuracy and speed of magnetic resonance frequency tracking in a rapidly varying field.

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PROBLEM DEFINITION

Optically pumped quantum M_X magnetometers (OPQMs) measure the frequency of forced oscillations of the total atomic magnetic moment transverse component [1]. In the OPQM, the atomic resonance signal arises when the frequency of a microwave field applied to a cell with alkali metal atoms coincides with the frequency of an atomic transition. In this case, the pumping radiation intensity transmitted through the cell becomes modulated, with the modulation frequency being equal to the applied microwave field and the phase of modulation being dependent on the offset of the microwave frequency from the atomic resonance frequency. This effect is central for detecting atomic resonances with optically pumped magnetometers, which offer a much higher sensitivity than their proton counterparts: one quantum of the atom-absorbed microwave field yields one photon having a 10^9 times higher energy.

OPQMs of the highest sensitivity use alkali metal (cesium, rubidium, or potassium) atoms as a working substance. Isotopes ^{133}Cs and ^{87}Rb used in cesium and rubidium magnetometers exhibit a small quadratic Zeeman splitting; therefore, in typical terrestrial fields, the resonance lines of the atom merge into one broad (about 20 nT wide) symmetric band whose shape and position depend on the pumping parameters, orientation of the device in the magnetic field, etc.

Isotope ^{39}K and especially isotope ^{41}K exhibit a much larger quadratic Zeeman splitting. In the terrestrial magnetic field, the resonance lines of the K atoms are well resolved and the linewidth may be decreased to 0.2 nT. That is why the absolute accuracy and long-term stability of potassium magnetometers are much higher than those of cesium and rubidium ones. The price for the improved characteristics is a complex resonance pattern consisting of several lines, which makes it necessary to apply computer-aided techniques of signal analysis and microwave resonance field frequency synthesis. In this work, we suggest a digital way of constructing a phase-locked loop for optically pumped quantum M_X magnetometers that provides a high accuracy and speed of identification both of a single line (cesium magnetometer in the terrestrial magnetic field) and of a complex structure consisting of very narrow lines (potassium magnetometer).

In terms of magnetic resonance excitation, OPQMs are categorized as self-excited devices, where the photocurrent amplifier is directly connected to the rf field coil by a feedback loop, and devices where the phase-locked loop (PLL) is directly tuned to the resonance. Magnetometers of either type are characterized by errors including parameter-drift-related shifts of the resonance itself and inaccuracies in measuring the position of the resonance line center. The basic source of inaccuracies in both types of M_X magnetometers is mistuning of the PLL, which causes a frequency offset of the output signal. Therefore, the phase of the applied microwave field and the phase of the magnetic moment precession signal recorded by the photodetector must be accurately matched.

Self-excited magnetometers, though simple in design, need a frequency meter capable of rapidly measuring the frequency of the inevitably noisy signal with an extremely high accuracy (for potassium OPQMs, the requirements are as follows: the frequency range 100–700 kHz and an accuracy of $(2–5) \times 10^{-9}$ for a measurement time of 0.1 s). The signal to be measured from PLL OPQMs is much less noisy. Moreover, if a digital frequency synthesizer is used as an oscillator in the PLL, a precision frequency meter becomes unneeded: in this case, measurement of the oscillator frequency is replaced by digitization of control sequences at the inputs of the synthesizer. Below, we will concentrate on PLL magnetometers. Here, the error signal controlling a tunable oscillator (a voltage-controlled oscillator in analog schemes or a digital frequency synthesizer in digital schemes) is separated out with a phase detector in which the reference voltage phase is set using a phase shifter so as to compensate for all phase shifts due to delays in the measuring channel and detector geometry (namely, the orientation of the pumping beam and rf field coil).

In this work, we describe an approach to constructing a digital phase-pocked loop that makes it possible to compensate for all the shifts mentioned above. For the theory of digital PLL construction, which is today well developed, see [2].

DIGITAL PHASE SHIFTER OPERATION

Let us estimate a desired accuracy of tuning the phase signal. Signal $S(t)$ due to a transition between magnetic sublevels in a microwave field of amplitude H_1 and frequency ω superposed on uniform magnetic field H_0 is described by the Bloch equation [3], the stationary solution to which in the complex form is given by

$$S(t) = \frac{A\omega_1}{Z^2}\frac{i+\delta Z}{1+\delta^2}e^{i(\omega t+\varphi)} = S_0 e^{i\omega t}. \tag{1}$$

Here, S_0 is the complex amplitude of the M_X resonance signal, $\delta = (\omega - \omega_0)/\Gamma Z$ is the reduced offset of microwave frequency ω from resonance frequency $\omega_0 = \gamma H_0$, φ is an extra shift of the resonance phase associated with phase shifts in the measuring channel and with the geometry of the experiment, A is the resonance signal amplitude, $2\Gamma = 2/T_2$ is the resonance width, $Z = (1 + \omega_1^2 T_1 T_2)$ is the resonance saturation factor, $\omega_1 = 1/2\gamma H_1$ is the Rabi frequency, γ is the gyromagnetic ratio for the working substance, and T_1 and T_2 are the times of longitudinal and transverse magnetic moment relaxation.

Suppose that we detect the signal with two identical phase detectors one of which (let it be detector x) is in phase with the microwave field and the other (detector y) is out of phase by 90°. Then, the output signals of phase detectors x and y averaged over characteristic time τ ($\tau \gg 1/\omega$) are expressed as

$$x(\delta, \varphi) = k\langle \mathrm{Re} S_0\rangle = k\frac{A\omega_1}{Z^2}\left[-\frac{1}{1+\delta^2}\sin\varphi + \frac{\delta Z}{1+\delta^2}\cos\varphi\right], \tag{2a}$$

$$y(\delta, \varphi) = k\langle \mathrm{Im} S_0\rangle = k\frac{A\omega_1}{Z^2}\left[\frac{\delta Z}{1+\delta^2}\sin\varphi + \frac{1}{1+\delta^2}\cos\varphi\right], \tag{2b}$$

where k is the transfer coefficient common to both detectors. At $\varphi = 0$, components $x(\delta, 0)$ and $y(\delta, 0)$ represent, respectively, a Lorentz dispersion contour with a zero value at a zero offset and a Lorentz absorption contour with a maximum at a zero offset. Signal component $x(\delta, 0)$ is used as an error signal, which the PLL tends to nullify. Component $y(\delta, 0)$ can be used as a signal amplitude indicator at a zero offset. To establish a correspondence between the frequency shift and phase shift in the closed PLL, we equate the right of (2a) to zero to obtain $\tan\varphi = \delta Z$. In other words, a phase shift by 45° introduces an error of about a half-linewidth. Typically, the half-linewidth in potassium OPQMs is 1 nT, and their ultimate accuracy, which is limited by the parametric-drift-related instability of M_X resonance, is on the order of 10 pT. Therefore, for the potential of the potassium OPQM to be realized completely, the phase offset must not exceed arctan(1/100) = 0.6°. However, the phase shift to be compensated for by the phase shifter may vary from 0° to 180° or more depending on the signal frequency, signal channel length, time delay of the signal in the photodetector, etc. Thus, the problem of constructing a controlled analog phase shifter capable of providing such parameters for the flat amplitude–frequency response in the frequency range 100–700 kHz is practically unsolvable.

Basically, a signal with any reference phase can be detected by digital methods provided that the rate of digital data processing is sufficiently high. Specifically, for a potassium OPQM in the terrestrial magnetic field, this means real-time signal digitization and processing with a rate of 10^7–10^8 cps, which also seems problematic even if high-end signal processors are applied.

We suggest a digital way of signal phase rotation (in synchronous detection schemes) through arbitrary angle α. It is based on the fact that, for any harmonic function

$$f(t) = (a+ib)e^{i\omega t} s e^{i(\omega t+\varphi)}$$

phase rotation through angle α and phase detection by square-law synchronous detectors x' and y' at frequency ω show the commutative property; i.e., these operations may be accomplished both in direct sequence,

$$\begin{aligned} x' &= k\langle f'(t)\cos\omega t\rangle, \\ y' &= k\langle f'(t)\sin\omega t\rangle, \end{aligned} \tag{3}$$

where $f'(t) = f(t)e^{i\alpha}$, and in reverse sequence. This is true if, after initial function $f(t)$ has been detected by square-law synchronous detectors x and y, phase rotation through angle α can be accomplished by calculating the linear combination

$$x' = x\cos\alpha - y\sin\alpha,$$
$$y' = x\sin\alpha + y\cos\alpha, \quad (4)$$

where $x = k\langle f(t)\cos\omega t\rangle$ and $y = k\langle f(t)\sin\omega t\rangle$. It is easy to check that transformations (3) and (4) yield the same result,

$$x' = k[a\cos\alpha - b\sin\alpha] = ks\cos(\varphi + \alpha),$$
$$y' = k[a\sin\alpha + b\cos\alpha] = ks\sin(\varphi + \alpha), \quad (5)$$

but transformation (4) does not require the phase rotation of the signal at frequency ω. Let us clarify this point by the example of the magnetic resonance signal. Expressions (2) describe the output voltages of two phase detectors in the absence of phase shifters. If these voltages were integrated with time constant τ, they are free of components at frequencies higher than $1/\tau$ and so can be digitized with a measurement period $\sim\tau$. For $\tau = 5$–10 ms, digitization can be made with a rate of (5–10) $\times 10^3$ cps using, e.g., a 10-bit ADC built in an 8-bit general-purpose microcontroller. Having digitized voltages $x(\delta, \varphi)$ and $y(\delta, \varphi)$, we, in accordance with (4), make up (now using digital means of the microcontroller) their linear combinations, introducing a new parameter, virtual phase angle ψ,

$$x'(\delta, \varphi, \psi) = x(\delta, \varphi)\cos\psi - y(\delta, \varphi)\sin\psi,$$
$$y'(\delta, \varphi, \psi) = x(\delta, \varphi)\sin\psi + y(\delta, \varphi)\cos\psi. \quad (6)$$

We obtain two new virtual signals, which can be conveniently assigned to the outputs of two virtual detectors x' an y'. As follows from (2) and (6),

$$x'(\delta, \varphi, \psi) = x(\delta, \varphi + \psi),$$
$$y'(\delta, \varphi, \psi) = y(\delta, \varphi + \psi). \quad (7)$$

Thus, we realized digitally (by software) an additional rotation of the signal phase through angle ψ. With phase ψ such that $\psi = -\varphi$, the output of virtual detector x' can be used as an error signal and that of detector y' has then the meaning of the signal amplitude.

Thus, our next goal is to set phase $\psi = -\varphi$ equal to $-\varphi$, i.e., to tune the virtual phase shifter. If phase shift φ is not known in advance, it can be estimated by the method of invariant mapping of the spin precession signal [4].

After the coarse adjustment of the phase, the system locks in resonance and then the phase shift is accurately measured by the modulation method. A change in the phase shift in response to a change in resonance frequency ω is taken into account in the form $\varphi(\omega) = \omega_0 + \omega T_d$, where φ_0 is the phase shift due to the field coil configuration and T_d is the effective delay of the signal in the signal channel.

The amount of error involved in such a method of phase shift compensation depends on the shift measurement inaccuracy (which, in turn, wholly depends on the magnetic field variation at the time of measurement) and the instability of the digital phase shifter. The latter depends on the instability of the phase relation between two outputs of the digital frequency synthesizer and on the (e.g., temperature-induced) instability of the relation between the transfer coefficients of detectors x and y. The absolute error of the digital phase shifter fabricated using advanced technologies can be reduced down to 0.1°.

EXPERIMENT

Following the concepts stated in the previous section, we fabricated and tested a potassium OPQM with a digitally controlled PLL, shown in Fig. 1. The PLL is free of analog tuning units: the functions of M_X resonance recognition and locking are performed by a microprocessor-controlled frequency synthesizer.

The basic components of the PLL are a signal amplifier, phase detectors x and y (implemented as analog multipliers with low-pass second-order filters), a microcontroller with ADCs at the inputs that is clocked by a stable quartz oscillator, and a digital frequency synthesizer with two square-law outputs phase-shifted strictly by 90°. The voltages from the square-law outputs are applied to the phase detectors as reference signals.

Initially, the synthesizer generates a microwave field at a far-from-resonance frequency. The variable component of the signal picked up from the detector is used to calibrate the zeros of the phase detectors. After calibration (lasting about 0.8 s), the processor starts searching for the resonance, i.e., scans the frequency of the synthesizer. The scan direction is selected such that the strongest (for a given configuration of the device) low-frequency resonance in the potassium atomic structure ($F = 2$, $m_F = 1 \rightleftharpoons 2$) is excited first. Scanning is continued until the variable component of the signal from the detector becomes appreciable. Then, the processor locks the frequency of the synthesizer. If the signal is lost, the processor searches for the resonance and locks it again. The input signal is sampled with a rate of 5120 Hz. After each sampling, the frequency of the synthesizer is adjusted at the same rate with the step proportional to the error signal at the output of virtual detector x'. The minimal step of the synthesizer in terms of the magnetic field strength is $\Delta H = 0.17$ pT. Significantly, the digital frequency synthesizer used in our work allows for continuous time variation of the phase of the generated voltage as the synthesized frequency varies.

Over one measurement cycle, $T = 0.1$ s, the registers of the processor accumulate and average 512 counts of the synthesizer frequency. The result of averaging is converted to a serial code and transmitted to the user computer along with data for the output voltages of the phase detectors, feedback state, etc.

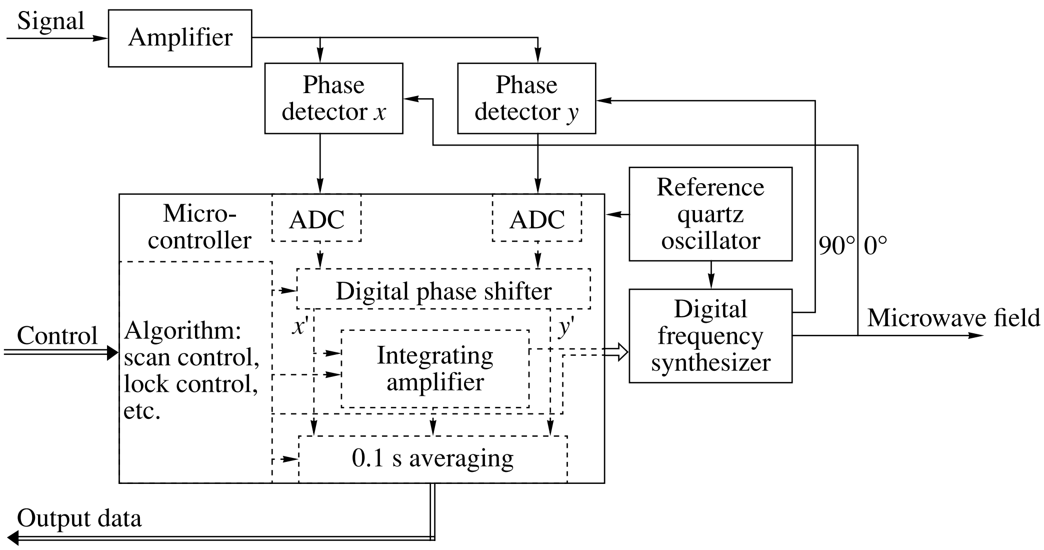

Fig. 1. Functional diagram of the digital phase-locked loop.

Sensitivity δH of the magnetometer for time t is basically limited by photocurrent shot noise intensity N recorded by the photodetector, $\delta H \sim N(t)/(A\gamma T_2)$. The effective resolving power of the system depends only on the shot noise intensity and remains independent of the step of the synthesizer for as long as the contribution of photocurrent short noise intensity δH measured for one measurement cycle exceeds step ΔH of the synthesizer. Under this condition, averaging of a discrete data array measured over time period T improves the resolving power in proportion to $\sqrt{T/t}$. Thus, the ultimate resolving power of the recording system related to the discreteness of the synthesizer's step equals $\Delta H/\sqrt{512} = 7$ fT for a measurement time of 0.1 s and does not limit the sensitivity of the magnetometer.

If necessary, the microcontroller slowly scans the synthesizer frequency about the resonance center to digitize the resonance line shape (Fig. 2) and then finely tune the phase. The curves in Fig. 2 are the outputs of virtual detectors x' and y' after tuning phase shift ψ as a result of scanning the microwave frequency in the stabilized field. A way of representing the resonance in the form of an ellipse on plane $x'y'$ is given in [3]. Such a representation makes it possible to measure the shift of the resonance phase in a rapidly varying field.

The PLL itself was implemented by software using the scheme of a proportional–integrating amplifier. In the feedback loop, the amplification coefficients of the amplifier operating in the proportional and integrating modes are specified by software. For given signal parameters, these coefficients control the speed and intrinsic noise of the PLL. The digital PLL was tested using an M_X resonance signal electronic simulator (a series circuit consisting of a quartz cavity and a controllable white noise generator) in a three-layer magnetic screen and in a magnetic field stabilizer. The tests with the signal simulator, where the white noise level was varied with the resonance parameters fixed, showed that the noise of the digital PLL depends on the input signal noise until the sensitivity becomes equal to 0.3 pT in terms of the variance for a measurement time of 0.1 s. The speed of the magnetometer with the digital PLL was tested in the magnetic screen. To this end, a variable magnetic field of amplitude 45 nT and frequency ranging from 0 to 100 Hz was imposed on the uniform magnetic field. At a count rate of 10 cps, the frequency band was tracked accurate to 3.5 Hz and the detectable magnetic field variation was 5000 nT/s or less. In searching for the resonance, the magnetic field scan rate was also equal to 5000 nT/s: it was shown that even such a scan rate provides resonance locking. The resolving power of the potassium OPQM with the digital PLL was tested in the magnetic field stabilizer. Figure 3 presents the Allan diagram of the magnetic field in the stabilizer. The variation is minimal, $\Delta H_{\min}$ =

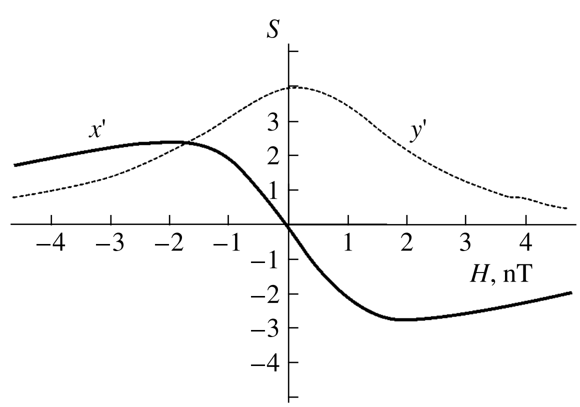

Fig. 2. M_X resonance line in the magnetic screen.

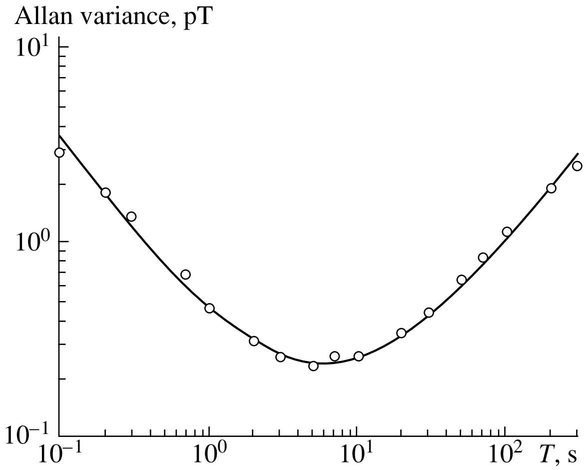

Fig. 3. Allan diagram of the magnetic field in the stabilizer.

0.23 pT in terms of the variance, for a measurement time of 5 s; for a measurement time of 0.1 s, ΔH = 3.3 pT (in terms of the variance). Of course, these values include the field variation in the stabilizer. Attention should be paid to the slope of the Allan diagram: it corresponds to the case when the amount of variation is in inverse proportion to the averaging time (rather than being in direct proportion to the square root of the averaging time, as it must be for Gaussian-type shot noise). This indicates the presence either of extra high-frequency noise in the PLL or of separate high-frequency components in the magnetic field spectrum. Specifically, the amplitude of the magnetic field variable component of frequency 50 Hz ranged from 200 to 250 pT (in terms of the variance) during the experiment and so might well contribute to the Allan variation over short measurement times.

CONCLUSIONS

The most attractive feature of digitally constructed schemes intended for M_X resonance frequency measurement in a variable field is that they are totally free of analog phase shifters and, therefore, offer a several orders of magnitude higher accuracy and long-term stability of the magnetometer. In addition, with such schemes, a high sensitivity and speed peculiar to the optically pumped quantum magnetometer may be shown in full measure.

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