ISSN 1063-7842, Technical Physics, 2008, Vol. 53, No. 5, pp. 646–654. © Pleiades Publishing, Ltd., 2008.
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EXPERIMENTAL INSTRUMENTS AND TECHNIQUE

Optimization of the Q Factor of the Magnetic M_x Resonance under Optical Pump Conditions

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Received June 28, 2007

Abstract—Optimization of parameters of the M_x resonance, which excited between Zeeman sublevels of a single HFS level of the ground state of an alkali metal in a vacuum cell in a circuit with a single beam for pumping and detection (M_x magnetometer circuit), has been carried out. A simple model taking into account all main factors controlling the resolving power of the M_x resonance (including spin-exchange broadening and absorption in a thick layer of the cell has been constructed. It is shown that the spin-resonance broadening of the resonance line is mainly determined by the requirements imposed on the optical thickness of the cell, which considerably restricts the realization of advantages of ultranarrow (<1 Hz) lines in magnetometry. The experiment confirming the efficiency of the model has been carried out.

PACS numbers: 07.55.Ge

DOI: 10.1134/S1063784208050198

INTRODUCTION

The advances made in quantum magnetometry in the last five decades resulted in the development of a number of magnetometric devices based on optical pumping [1, 2] and optical detection [3] of magnetic resonance. The wide-range cesium magnetometer [4] and a more complicated but more precise potassium magnetometer operating on a single line [5–7] are apparently the most promising instruments for measuring the magnitude of terrestrial magnetic fields (20–80 μT). Both these instruments belong to the class of M_x magnetometers since beam modulation of optical pump by the transverse component of the rotating magnetic moment during the interaction with a resonant radio-frequency field (M_x resonance) is used as the observed signal in the circuits of these instruments. The sensitivity of such instruments is entirely controlled by the magnetic resonance parameters and quantum noise that cannot be eliminated in principle.

This study aims at optimization of the parameters of the M_x resonance, which is excited between Zeeman sublevels of the same HFS level of the ground state of an alkali metal in a vacuum cell in a circuit with a single beam for pumping and detection. The main factors determining the resolving power of the M_x resonance, including spin-exchange broadening and absorption in a thick layer of the cell, are taken into account in optimization.

The exact solution of the problem of optimization of resonance parameters in a multilayer system involves considerable difficulties (see, for example, [8]) since it necessitates the inclusion of the dependence of absorption in the cell as well as of optical and collision broadening on the degree of polarization of the substance, which is in turn determined by pumping conditions. We will consider optical pumping in a cell with an antirelaxation coating, in which level "mixing" in the excited state is negligibly small. In this case, the problem can be solved exactly only using numerical methods. However, we set out to construct a simple analytical model having a clear physical meaning and capable of providing correct quantitative predictions. It will be shown that such a model can be constructed in the framework of the following approximations.

(1) Magnetic resonance is described by the Bloch equations [9] for a two-level system. This approximation is justified for

(i) the K atom in high (terrestrial) magnetic fields since the high degree of isolation of its resonance line makes it possible to consider magnetic resonance only between two energy levels of the "working" transition ($F = 2, m_F = 1 \longrightarrow F = 2, m_F = 2$), provided that the total population of these levels is smaller than unity. Numerical analysis shows that the difference in the occupancies of this pair of levels for the optimal intensity of pump light amounts to approximately 0.5, while the sum (denoted by k_{pump}) is $k_{\text{pump}} \approx 0.75$;

(ii) for all alkali metals (including Cs) in the opposite case of weak magnetic fields, in which quadratic Zeeman splitting is smaller than the intrinsic linewidth (this case is realized for cesium when the magnetic field induction does not exceed 20 μT).

(2) Spin-exchange broadening of the resonance line is assumed to be independent of the degree of polarization of the working substance

(3) Pumping and detection of resonance is performed by a single beam.

(4) Lifetime T_1 of atoms, which is much longer than time of flight τ_{fl} from wall to wall in the cell ($T_1 \sim 10^4\tau_{fl}$), leads to complete effective averaging of the pump light intensity and to a uniform distribution of population of each level over the volume.

(5) For the same reason, pumping of all atoms in the cell by monochromatic radiation tuned to the center of the Doppler absorption line is effectively performed (during the characteristic pump time, each atom has time to be many times in the group of velocities, for which resonance between the pump line and the Lorentz profile of atom absorption is ensured).

(6) The width of the pump spectrum is assumed to be small as compared to the resonance absorption line.

Parameters of the M_x resonance will be optimized in accordance with the criterion of maximal sensitivity of the quantum discriminator. We will not consider here in detail fundamental long-term shifts and systematic errors of quantum discriminator, stipulating that all of the above effects are proportional to the linewidth of the M_x resonance [7] and can be minimized by reducing collision and optical broadening of the resonance line to values ensuring the required short-term sensitivity.

1. THEORETICAL ANALYSIS

The main mechanisms of magnetic moment relaxation, which control the magnetic resonance width, were listed in [7] and included relaxation during collisions with cell walls, spin-exchange processes, relaxation in collisions with buffer gas atoms, nonuniformity of an external magnetic field, relaxation induced by pumping light, and broadening by an rf field.

Collisions with cell walls destroying polarization in each such collision (i.e., during the time of flight on the order of $\tau_{fl} \approx 10^{-5}$–$10^{-4}$ s) obviously dominate in a cell without a buffer gas and coating. The most effective method for preserving polarization (especially in a nonuniform magnetic field) is the deposition of preserving spin coating on the inner wall of the cell. Probably the best results in the development of paraffin coating deposition technique were obtained by Balabas et al. [10–12] belonging to the Alexandrov group. These studies were a continuation of the research carried out by Bouchiat and Brossel [13–15], who studied spin-relaxation processes in optically polarized Rb atoms on a paraffin surface.

Summarizing the results of these studies, we assume that the magnetic resonance width (disregarding broadening by an ac magnetic field) in a paraffin-coated cell with a reservoir or a spout containing a spare volume of the metal can be presented as the sum of several components:

$$\Gamma = [(\Gamma_{wall} + \Gamma_{hole}) + \Gamma_{coll}] + \Gamma_{light}. \quad (1)$$

Here, Γ_{wall} is the contribution of relaxation at the cell wall, $\Gamma_{wall} = G_w/\tau_{fl}$, where $C_w = C_w(T)$ is the probability of electron spin relaxation during collision with the coating; $\tau_{fl} = \langle l_{fl}\rangle/\langle v\rangle$ is the mean time of flight from wall to wall; and $\langle l_{fl}\rangle$ is the mean free path between collisions with the wall. For a spherical cell of diameter D, $\langle l_{fl}\rangle = k_{form}D$ and $k_{form} \approx 0.7$, while for a cylindrical cell of diameter D and length $L = lD$ ($l \gg 1$), we have $\langle l_{fl}\rangle = k_{form}\sqrt{DL}$ and $k_{form} \approx 0.5$. Quantity $\langle v\rangle = \sqrt{\frac{8}{\pi}\frac{RT}{M}}$ is the mean velocity of atoms, T is the temperature, M is the molar mass, R is the universal gas constant, and Γ_{hole} is the rate of absorption of polarized atoms by the reservoir. In the simplest case when the reservoir is separated from the cell by a diaphragm of area s, we have $\Gamma_{hole} = s/(S\tau_{fl})$, where S is the area of the inner surface of the cell and Γ_{coll} is the line broadening due to atomic collisions (spin-exchange or collision broadening), which is proportional to the concentration n of atomic vapor: $\Gamma_{coll} = \sigma_{ex}n\langle v_{rel}\rangle$, where σ_{ex} is the cross section of the spin-exchange process and $v_{rel} = \sqrt{2}\langle v\rangle$ is the relative velocity of colliding particles, and Γ_{light} is the optical broadening proportional to intensity I_{ph} of photon pumping.

The brackets in Eq. (1) are introduced to group the terms of similar origin. For example, the first three terms are the dark linewidth $\Gamma_d = \Gamma_{wall} + \Gamma_{hole} + \Gamma_{coll}$. The first two terms in Eq. (1) are determined by the cell structure and the quality of coating and are known as the natural resonance width $\Gamma_0 = \Gamma_{wall} + \Gamma_{hole}$ in the cell:

$$\Gamma_0 = k_R\frac{\langle v\rangle}{\sqrt{lD}},$$
$$k_R = \frac{1}{k_{form}}\left[C_w(T) + \frac{s}{S}\right]. \quad (2)$$

Relaxation associated with the departure of atoms to the reservoir with a metal drop is determined by area s. Henceforth, we will assume that the diaphragm is small enough to disregard the second term in k_R; in this case, the natural resonance width in the cell is inversely proportional to its diameter and the dark width Γ_d is proportional to vapor concentration n. The temperature dependence of dark width Γ_d of the M_x resonance in the potassium cell given in [7, 16] shows that this condition is indeed satisfied to within ±5% in the temperature range T = 20–50°C.

We can pass to equations for a spherical cell of diameter D assuming that $l = 1$.

We can estimate the limiting (i.e., restricted by fundamental physical factors) sensitivity of the M_x resonance by calculating the minimal variation of the magnetic field, which can generally be detected in measuring in the frequency band Δf. Under the condition of predominance of Schottky noise of pump light over other noise, such a variation can be expressed as

$$\delta B_{\min}^{\text{light}} = \frac{1}{\gamma}\frac{\rho_N}{(dS(\omega)/d\omega)|_{\Delta\omega=0}}\sqrt{\Delta f} = \frac{1}{Q}\sqrt{\Delta f}. \quad (3)$$

Here, γ is the gyromagnetic ratio, $\Delta\omega = \omega_0 - \omega$, ω_0 is the magnetic resonance frequency, ω is the frequency of the resonance rf field, ρ_N is the spectral density of Schottky noise in the light being detected, $S(\omega)$ is the amplitude of the M_x resonance signal synphase to the rf field, and $Q = (\gamma/\rho_N)dS(\omega)/d\omega|_{\Delta\omega=0}$ is the quality factor of the resonance [17].

The expressions for the amplitude as a function of the pump light intensity are given in [18, 19]; the need for optimizing the Q factor from the intensity of light was first indicated in [20]. It was shown in [21] (as applied to the problem of optimization of sensitivity of the M_Z frequency discriminator) that light intensity I_{opt} ensuring a 3–4-fold broadening of the dark resonance line is optimal if the Schottky noise of the pump radiation dominates over other noise. We will use the approach adopted in [21] and write the steady-state solution to the Bloch equation for the transverse magnetization component u (used in M_x-type magnetometers) in the rotating coordinate system:

$$u(\Delta\omega) = M_0\frac{\Delta\omega\Omega T_2^2}{1 + (\Delta\omega T_2)^2 + \Omega^2 T_1 T_2}. \quad (4)$$

Here, $M_0 \sim \Gamma_{\text{light}}/\Gamma$ is the longitudinal magnetization induced by the circular component of pump light along the magnetic field, $\Omega = \gamma B_1$ is the Rabi frequency of resonant field B_1, and T_1 and T_2 are the effective longitudinal and transverse relaxation times taking into account optical pump processes. This solution is valid when the magnetic resonance frequency ω_0 is much higher than characteristic relaxation rate $1/T_1$, $1/T_2$ (we will henceforth assume that $T_1 = T_2 = 1/\Gamma$).

Traditionally, the optimization of the Q factor begins with the choice of rf field amplitude Ω; for this purpose, the signal steepness $dS(\omega)/d\omega$ at the center of the resonance is calculated and then its maximum value is found from Ω ($\Omega_{\text{opt}} = \Gamma$). Steepness U of a signal optimized from Ω at the line center $S = S(0)$ is given by

$$\left.\frac{dS(\Delta\omega)}{d\Delta\omega}\right|_{\Delta\omega=0} = \frac{S}{2\Gamma}. \quad (5)$$

Let the optical scheme be axisymmetric and, accordingly, the cell have the shape of a cylinder of diameter D and length $L = lD$, whose axis is oriented parallel to the pump beam. Absorption of light of intensity I_{ph} in a medium of length L is described by the Bourguer-Lambert law $I_{\text{ph}}(L) = I_{\text{ph}}(0)\exp(-\kappa L)$, where κ is the coefficient of linear absorption of the medium, which is proportional to number density n of atoms: $\kappa = \kappa_\nu n$. According to [16], for pumping by radiation with frequency ν coinciding with the center of the Doppler line, we have

$$k_\nu = 2\sqrt{\pi\ln 2}\frac{e^2}{mc}\frac{1}{\Delta_D} f\delta_{\text{HFS}} = 2.493\times10^{-2}[\text{cm}^2\ \text{Hz}]\, f\delta_{\text{HFS}}/\Delta_D, \quad (6)$$

where e and m are the electron charge and mass, c is the velocity of light,

$$\Delta_D = 2\sqrt{\ln 2}\frac{1}{\lambda}\sqrt{\frac{2RT}{M}}$$

is the Doppler linewidth for an atom with molar mass M, λ is the wavelength, T is the temperature, f is the transition oscillator strength, and

$$\delta_{\text{HFS}} = \frac{\sum_j I_j \exp\left[-4\left(\frac{\nu_j - \nu}{\Delta_D}\right)^2 \ln 2\right]}{\sum_j I_j}$$

is the factor taking into account the presence of hyperfine transitions in the absorption spectra, which are characterized by intensities I_j and frequencies ν_j.

For pumping a potassium atom ($f = 0.35$, $\delta_{\text{HFS}} = 0.78$) by a laser tuned at the optical transition center, we obtain $k_\nu = 8.82\times10^{-12}$ cm^2 at temperature $T = 300$ K. In the case flashlamp pumping of cesium ($f = 0.39$) with four hyperfine lines, we obtain $k_\nu = 2.70\times10^{-11}$ cm^2 at the same temperature.

It should be noted that coefficient k_ν in the general case depends on the overlap integral of normalized pump ($\sigma_p(\nu)$) and absorption ($\sigma_a(\nu)$) contours: $\psi = \int\sigma_p(\nu)\sigma_a(\nu)d\nu$. Here and below, we disregard the temperature dependence of k_ν since the change in the mean velocity does not exceed 5% in the entire working temperature range 30–60°C (for K and Cs).

In accordance with formulas (1) and (2), we have

$$\Gamma_d = \Gamma_0 + \Gamma_{\text{coll}} = k_R(1 + \alpha x)\frac{\langle v\rangle}{\sqrt{lD}}, \quad (7)$$

where $x = \kappa L = k_\nu nL$ is the optical thickness of the cell and

$$\alpha \equiv \frac{\sqrt{2}\sigma_{\text{ex}}}{k_R k_\nu\sqrt{l}}$$

is a coefficient independent of n, $\langle v\rangle$, and D.

Coefficient α connects dimensionless collision broadening Γ_{coll} and optical thickness x of the cell

$$\frac{\Gamma_{\text{coll}}}{\Gamma_0} = \alpha x. \quad (8)$$

The value of α can be determined experimentally. For example, using the dependence of dark linewidth Γ_d on optical thickness x for a potassium cell of diameter

$d = 15$ cm given in [7, 16], we find that $\alpha = 32 \pm 8$ for this cell.

In accordance with relation (8), parameter α for a spherical cell of diameter D can be expressed as the product of atomic constants by the linewidth and the cell diameter:

$$\alpha = \frac{\sqrt{2}\langle v\rangle\sigma_{ex}}{k_v}\frac{1}{D\Gamma_0} = k_\alpha\frac{1}{D\Gamma_0} \quad (9)$$

(it should be noted that $D\Gamma_0 = \text{const}$ in accordance with relation (2)). For potassium with laser pumping, we have $k_\alpha \approx 65$ cm/s. Flashlamp pumping is characterized by a much lower efficiency, and values of k_α and α may increase in this case. For cesium in a weak magnetic field, we have $k_\alpha \approx 25$ cm/s for the maximum lamp pumping efficiency.

Let us introduce the dimensionless pump intensity $I = \Gamma_{light}/\Gamma_d$, equal to the ratio of light broadening to the dark width. Then it follows from relations (1)–(7) that

$$\Gamma = \Gamma_d + \Gamma_{light} = k_R(1 + I)(1 + \alpha x)\frac{\langle v\rangle}{\sqrt{lD}}. \quad (10)$$

Let us consider a region of the cell with unit area, length L along the pump beam and, accordingly, optical thickness $x = \kappa L = k_v nL$. We denote by I_{ph} the intensity of resonant pump radiation at the inlet to the cell. Using the expression for the pump rate (see p. 178 in [22]), we can connect light broadening Γ_{light} with intensity:

$$\Gamma_{light} = 2\frac{k_v}{h\nu}I_{ph}^{mean}. \quad (11)$$

In this case, averaging over the cell length along the beam gives

$$I = \frac{\Gamma_{light}}{\Gamma_d} = 2\frac{k_v}{h\nu}\frac{I_{ph}}{\Gamma_d}\frac{1}{x}\int_0^x e^{-x'}dx' = 2\frac{k_v}{h\nu}\left(\frac{I_{ph}}{(1 + \alpha x)\Gamma_0}\right)\left(\frac{1 - e^{-x}}{x}\right). \quad (12)$$

Amplitude S_0 of the M_x resonance signal in the region with the unit cross-sectional area under investigation is proportional to

(i) the degree of polarization of the medium $k_{pump}\Gamma_{light}/\Gamma = k_{pump}I\Gamma_d/\Gamma$;

(ii) the number of atoms in this region, $N_{at} = nL = x/k_v$; and

(iii) the efficiency of optical detection of the resonance, which in turn is proportional to intensity $I_{ph}e^{-x}$ of light passing through the cell, effectiveness k_{read} of "reading" of the signal by light, and quantum yield k_{det} of the detector. According to [23], $k_{read} = 1/(12\sqrt{2}) \approx 0.059$ for an isolated potassium line $F = 2, m_F = 1\rangle \longrightarrow F = 2, m_F = 2\rangle$ in the case when light forms an angle of 45° with the magnetic field. For a continuous cesium line under the same conditions, numerical calculations give $k_{read} \approx 0.0128$.

The total amplitude S of the signal can be obtained by integrating over the cross-sectional area of the cell, Substituting the values of Γ and Γ_d from relations (2) and (10), we obtain

$$S = k_{pump}k_{read}k_{det}\frac{\pi}{4}\frac{I^2}{1 + I}\frac{D^2}{k_v}\Gamma_d\frac{x^2}{1 - e^{-x}}e^{-x}. \quad (13)$$

The signal steepness can be obtained using relations (13) and (10):

$$\frac{S}{\Gamma} = k_{pump}k_{read}k_{det}\frac{\pi}{4}\frac{D^2}{k_v}\frac{I^2}{(1 + I)^2}\frac{x^2}{e^x - 1}. \quad (14)$$

The spectral density of the Schottky noise is proportional to the root of the total intensity of light transmitted through the cell, integrated over the cell area. Expressing the total intensity I_{ph}/ψ of light at the cell entrance in terms of the averaged dimensionless intensity I (12), we obtain

$$\rho_N = \left(\frac{\pi}{4}\frac{k_{det}}{\psi}\frac{k_R}{k_v}I\langle v\rangle\frac{D}{\sqrt{l}}(1 + \alpha x)\frac{x}{e^x - 1}\right)^{1/2}. \quad (15)$$

Finally, having expressed k_R in terms of α, we obtain the expression for the dependence of the Q factor on the pump intensity in the unit frequency band,

$$Q(\alpha, x, V, I) = \sqrt{\psi k_{det}}k_{pump}k_{read}Q_1(V)Q_2(I)Q_3(\alpha, x), \quad (16)$$

where

$$Q_1(V) = \left(\frac{V}{\sqrt{2}\sigma_{ex}\langle v\rangle}\right)^{1/2},$$

$$Q_2(I) = \frac{I^{3/2}}{(1 + I)^2},$$

$$Q_3(\alpha, x) = x(e^x - 1)^{-1/2}\left(1 + \frac{1}{\alpha x}\right)^{-1/2},$$

and V is the cell volume.

The dependence of expression (16) on I can be reduced to factor $Q_2(I)$, which coincides with that obtained in [20] (taking into account the frequency band).

The dependence of the signal intensity, steepness, and Q factor on the pump intensity is shown in Fig. 1. It is interesting to note that, in accordance with relation (14), signal steepness S/Γ for given values of I and x does not depend explicitly on the resonance line width and, hence, on the properties of coating. The entire dependence of the Q factor on the linewidth is determined by the contribution of noise factor (15): pumping of a broad line requires a higher pump power, which leads to an increase in the Schottky noise of light.

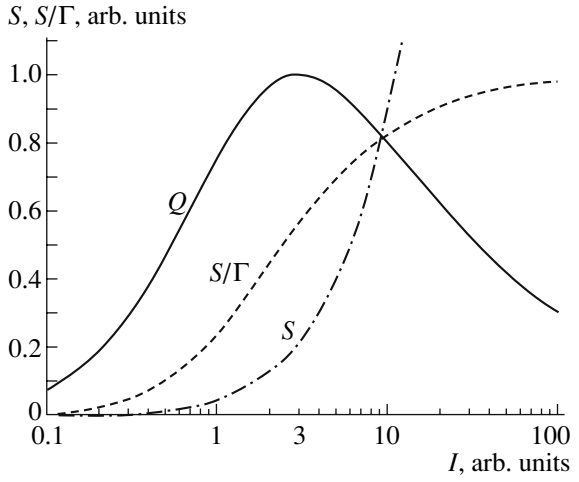

Fig. 1. Dependence of signal S, steepness S/Γ, and magnetic resonance Q factor on light-induced broadening $I = \Gamma_{light}/\Gamma_d$.

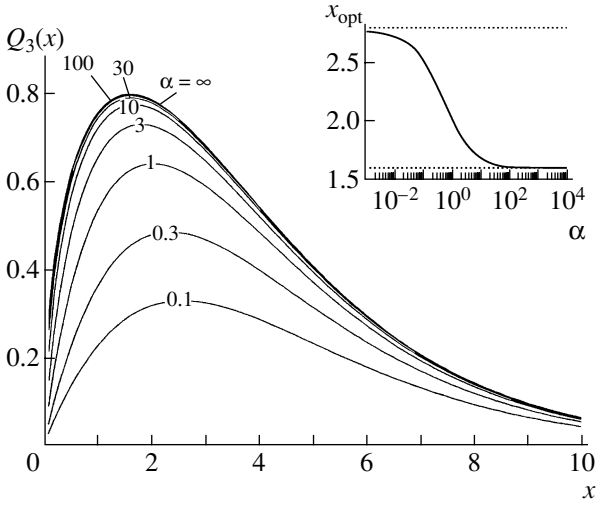

Fig. 2. Dependence of the Q factor on optical thickness x of the cell for various values of α. The inset shows the dependence of the optimal optical thickness x_{opt} of the cell on α.

Only the last factor $Q_3(\alpha, x)$ in relation (16) depends on optical thickness x; this dependence is shown in Fig. 2. The position x_{opt} of the optimal Q factor on the cell optical thickness axis (and, hence, of the vapor concentration axis) is obviously controlled exclusively by parameter α. It can be approximated by the following dependence (see inset to Fig. 2):

$$x_{opt} \approx 1.59 + 1.23(1 + \alpha/2)^{-0.91}. \quad (17)$$

The range of optimal values x_{opt} lies in the interval of $x = 1.59$–2.82. It can be seen from Fig. 2 that the optimal optical thickness of the cell for $\alpha > 10$ lies in the range $x_{opt} = 1.6$–1.7, which corresponds to attenuation of resonance light to approximately one-fifth of its initial value. It should be emphasized that all the above relations presume the presence of pumping and optimal resonance rf field. Optical thickness x'_{opt} measured in zero rf field turns out to be smaller than x_{opt}.

It is significant that the dependence of Q on α and, hence, on the properties of cell coating can be disregarded for $\alpha x \gg 1$ (this condition is observed practically for all cells with a small natural width). It follows from relations (16) and (17) that the spin-exchange line broadening must be defined by the expression

$$\Gamma_{coll}^{opt} \geq 1.6\frac{\sqrt{2}\sigma_{ex}}{k_v}\frac{\langle v\rangle}{L} \quad (18)$$

for any (even very small) value of natural linewidth Γ_0. For ^{39}K at $T = 300$ K, in the case of pumping by the D_1 line for a cell 15 cm in diameter, we obtain (Fig. 3) $\Gamma_{coll}^{opt}/2\pi = 1.1$ Hz; for Cs in a cell 1 cm in diameter, in the case of flashlight pumping ($\psi \approx 0.5$), we have $\Gamma_{coll}^{opt} = 12.8$ Hz. Consequently, spin-exchange broadening for the optimal optical thickness of the cell does not permit one to take advantage of ultranarrow lines observed in a cell with coating.

Expressions (3) and (16) make it possible to estimate the maximal attainable sensitivity δB_{min} of a quantum M_x sensor: for optimal values of I and x, we have $Q_{2max}Q_{3max} = 0.325 \times 0.805 = 0.262$; consequently, for optimal pump parameters, we have

$$Q_{max} = K_{opt}(\psi k_{det})^{1/2}V^{1/2}T^{-1}Hz^{1/2}, \quad (19)$$

where $K_{opt} = 1.84 \times 10^{13}$ cm$^{-3/2}$ for potassium and 1.42×10^{12} cm$^{-3/2}$ for cesium.

Let us evaluate δB_{min} for the experimental conditions described in [7]. In accordance with relation (19),

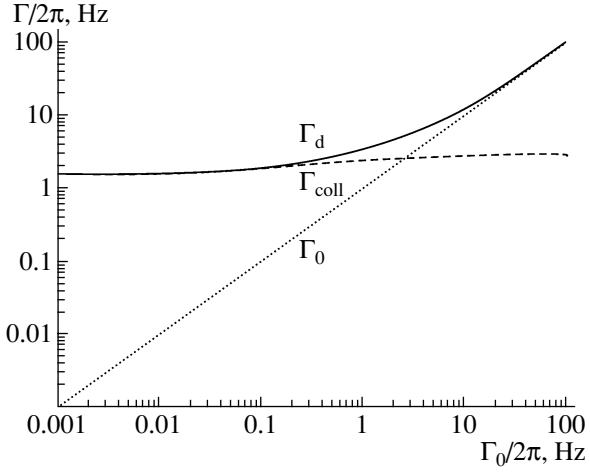

Fig. 3. Dependence of optimal collision linewidth Γ_{coll} and dark linewidth Γ_d on natural width Γ_0 for a potassium cell of diameter 15 cm.

$\delta B_{\min} = 1.6 \times 10^{-15}$ T Hz$^{-1/2}$ for potassium in a cell 15 cm in diameter and for detection coefficient $k_{\text{det}} = 0.35$ (here, k_{det} contains the filling factor of a multistrand optical fiber). Comparing this value with limiting sensitivity $\delta B_{\min} = (1.8–3.6) \times 10^{-15}$ T Hz$^{-1/2}$ for laser pumping experimentally measured in [7], we observe that our model can be used not only for describing the dependences of the Q factor on the pump parameters qualitatively (this will be demonstrated in the next section), but also to obtain correct quantitative predictions.

The physical meaning of the coefficients appearing in relation (16) becomes clear if we express Q_1 in terms of the number of atoms in the cell:

$$Q_1(V) = \gamma\left(\frac{V}{\sqrt{2}\sigma_{\text{ex}}\langle v\rangle}\right)^{1/2} = \gamma\left(\frac{N_{\text{at}}}{\Gamma_{\text{coll}}}\right)^{1/2}. \qquad (20)$$

It follows from this relation that the Q factor is proportional to the square root of the number of atoms in the cell. We assumed so far that Schottky noise of photocurrent is a factor limiting the sensitivity of a quantum discriminator with optical pumping. However, atomic quantum noise is the most important factor limiting the sensitivity of any quantum magnetometer. The limiting sensitivity (over time period τ) restricted by atomic noise is given in [24]:

$$\delta B^{\text{at}}_{\min} = \frac{1}{\gamma}\frac{1}{\sqrt{\langle N_{\text{at}}\rangle T_2 \tau}}. \qquad (21)$$

It follows from this estimate that coefficient Q_1 appearing in relation (20) is equal to Q factor $Q^{\text{at}}_{\min}$ limited by atomic noise to within a factor of $\sqrt{2\pi}$; consequently, we can write

$$Q(\alpha, x, V, I) = [\sqrt{2\pi k_{\text{det}}\psi}\, k_{\text{pump}} k_{\text{read}} Q_2(I) Q_3(\alpha, x)] Q^{\text{at}}_{\max}. \qquad (22)$$

Thus, the limiting Q factor is determined by quantum atomic noise and the efficiency of optical pumping/detection of the resonance. For a potassium discriminator we have $Q_{\max}/Q^{\text{at}}_{\min} \approx 0.025$; i.e., light noise exceeds quantum atomic noise by 1.5 orders of magnitude due to the fact that the number of test beam photons interacting with atoms per unit time is small as compared to the total number of photons in the beam.

In addition to the above arguments, one more circumstance limiting the advantage of using narrow resonance lines also exists. Although the response of the M_x circuit of the magnetometer is not limited by the resonance linewidth, the operation speed of any quantum magnetometer is limited by general indeterminacy in measuring the frequency of a harmonic signal containing noise over a given time interval, which is determined by the Cramér–Rao relation [25].

The expression for the lower limit of accuracy in measuring frequency for a signal containing an additive correction in the form of white noise is given in [26, 27]. Recalculating to the scale of magnetic field B, we can express this indeterminacy in the form

$$\delta B^{\text{meas}}_{\min} = \frac{1}{\gamma}\frac{\sqrt{3}}{\pi}\frac{\rho_N}{S}\frac{1}{\tau^{3/2}}. \qquad (23)$$

Assuming that minimal variations (3) and (23) are noncorrelated, we obtain the following estimate of the total indeterminacy in the field being measured:

$$\delta B = \sqrt{(\delta B^{\text{meas}}_{\min})^2 + (\delta B^{\text{light}}_{\min})^2} = \frac{1}{\gamma}\frac{\rho_N}{S}\frac{1}{\tau^{1/2}}\left(\left(\frac{\sqrt{3}}{\pi}\frac{1}{\tau}\right)^2 + \left(\frac{\sqrt{2}}{\sqrt{\pi}}\frac{1}{T_2}\right)^2\right)^{1/2}. \qquad (24)$$

The contributions of two variations in (24) become equal for $\tau/T_2 = (3/2\pi)^{1/2} \approx 0.7$.

Thus, for measuring time $\tau \gg T_2$, the limiting sensitivity of the magnetometer obeys the root dependence on the measuring time, while for $\tau \ll T_2$, the limiting sensitivity of the magnetometer deteriorates with decreasing measuring time in proportion to $\tau^{-3/2}$ in accordance with relation (23).

Figure 4 shows an example of calculation of the time dependence of limiting sensitivity of a quantum magnetometer. The calculations were made from the results of measurements of the sensitivity of a potassium sensor with laser pumping and an ultranarrow resonance line, which is limited by Schottky noise of photocurrent [7]. It can be seen that, as expected, light noise exceeds atomic noise by 1.5 orders of magnitude, while the Cramér–Rao relation limits the sensitivity of the instrument even for time periods $\tau < 0.1$ s.

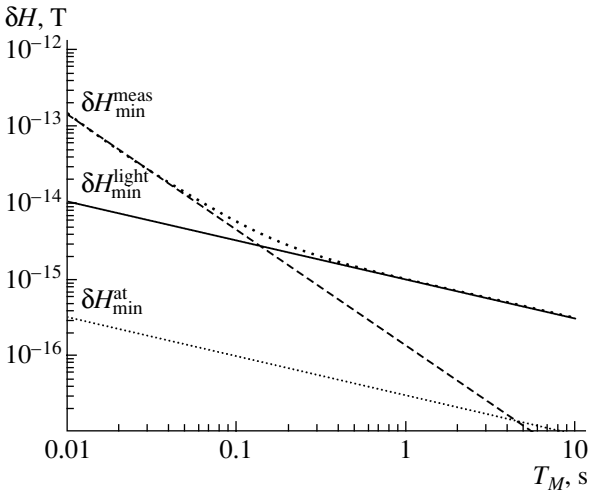

Fig. 4. Limiting sensitivity of a magnetometer with laser pumping calculated by formulas (23) and (24) using experimental results obtained in [26].

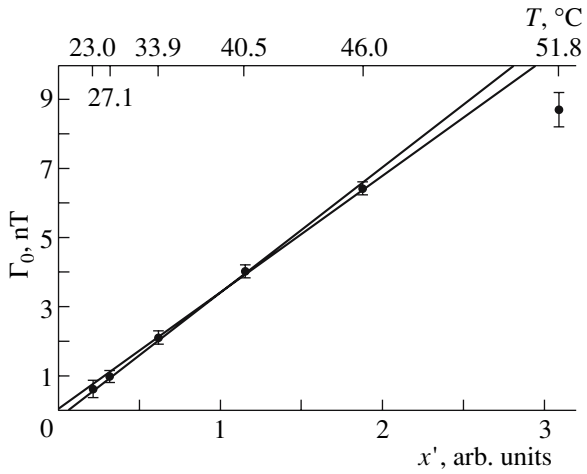

Fig. 5. Dependence of the HWHM dark linewidth of the M_x resonance in a cesium cell on the optical density of the cell (lower scale) and its temperature (lower scale).

Contemporary problems in studying the magnetic field of the Earth require a high data flow rate (ten and more counts in 1 s). This means that in solving such problems, it is inexpedient to reduce the linewidth to values smaller than 1 Hz; by the way, this is in good agreement with the results of above optimization.

The readings of discriminators receive significant contributions from various slow drifts (usually described by dependences ~ $1/f$, ~ $1/f^{1/2}$, etc.) over time periods on the order of ten seconds and longer. Analysis of such effects is beyond the scope of this paper; here, we only note an important feature inherent in quantum discriminators: their drifts over long time periods cannot be much larger than the resonance linewidth. Thus, narrowing of the resonance line is the main trend in the problems requiring higher accuracy of measurements over long periods of time. Depending on the problem, a choice has to be made between sensitivity and accuracy of measurements. For example, taking into account the fact that all optical shifts of the resonance line, as well as parametric shifts associated with them, are proportional to pump intensity, the working intensity on the gently sloping segment of the curve describing the dependence of Q factor on the intensity of pump light should be chosen lower than $I_{opt} = 3$. A decrease in intensity by a factor of 2 as compared to I_{opt} corresponds to a decrease in the Q factor by less than 10%. The optical thickness of the cell ensuring a collision broadening slightly smaller than the optimal value can be chosen in the same way. Further, it follows from (16)–(18) that an increase in the length of a cylindrical cell for its constant diameter will make it possible to linearly reduce the resonance linewidth for a constant optical thickness. In this way, not only the Q factor can be increased, but also the shifts of the discriminator can be reduced.

2. EXPERIMENT: OPTIMIZATION OF MAGNETIC RESONANCE PARAMETERS IN A CAESIUM CELL

It was noted above that approximations adopted here correctly describe the actual situation for

(i) potassium in the field of the Earth;

(ii) all alkali metals (including Cs) in weak magnetic fields.

The latter case can easily be realized in laboratory conditions and was chosen to verify the model constructed here. Magnetic resonance parameters in a cell were analyzed for optimizing the Q factor of the resonance in a magnetic screen in a field of 3300 nT. The field in the screen was stabilized with the help of a cesium M_Z sensor. The small cell diameter (~10 mm) made it possible to obtain small optical thicknesses at room temperature.

The parameters of M_x resonance were analyzed as follows. In the working temperature range of the cell (20–52°C), 10–12 points T_i ensuring a uniform distribution of measurements on the cesium vapor scale were selected. The vapor density was calculated by the formulas given in handbook [28]. At each point T_i, preliminary optimization of the rf field and scanning of resonance frequency in a neighborhood of ±20 nT from the center of the resonance line was carried out for 8–12 values of pump light intensity I_{ph}; the amplitude of resonance, its width Γ and the extent of broadening by the rf field were measured. The effective optical thickness x' of the cell relative to resonance line for the D_1 line in zero resonance rf field was determined from the measurement of the dependence of the pump light intensity on the temperature of the cell.

Processing of the first batch of measurements over a wide temperature range showed that the dark linewidth linearly depends on the rated density of Cs vapor only at $T < 50$°C. At higher temperatures, the resonance width of the line and its amplitude sharply decreased and the dependence of $\Gamma_d(T)$ on past history was observed. Such a behavior can be explained only by anomalies in the properties of the paraffin coating (e.g., the phase transition in the light fraction of paraffin). All further measurements were performed in the temperature range 23–52°C.

The result of measurements of the $\Gamma_0(x')$ dependence is presented in Fig. 5. It can be seen that the natural resonance width Γ_0 in the cell does not exceed 0.07 nT and $d\Gamma_{coll}/dx' = 3.49 \pm 0.12$ nT. As a result of approximation of the dependence of light broadening on x' by expression (12), ratio $x/x' = 2.10 \pm 0.35$ was calculated and, hence $\alpha = (d\Gamma_{coll}/dx)/\Gamma_0 = (25–\infty)$. In accordance with formula (16), the contribution from coefficient α lying in this limits to the Q factor for $x > 1$ does not exceed 2%.

Figure 6 shows the dependence of the limiting Q factor on light-induced broadening at various temperatures (experimental dependences and the result of their

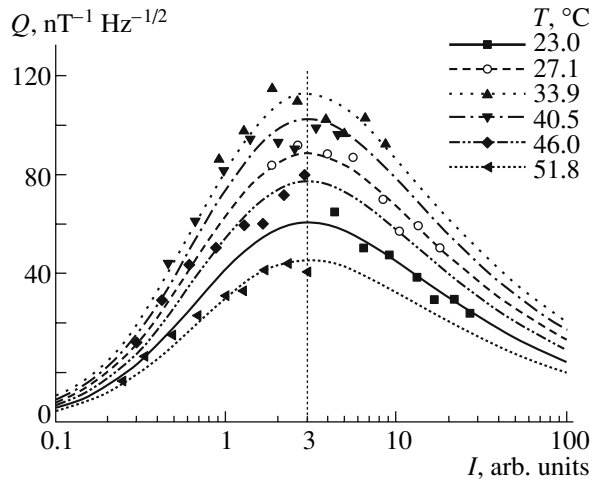

Fig. 6. Dependence of the Q factor in a cesium cell on light broadening for various values of cell temperature. Symbols correspond to experiment, and the curves are the results of approximation by formula (16).

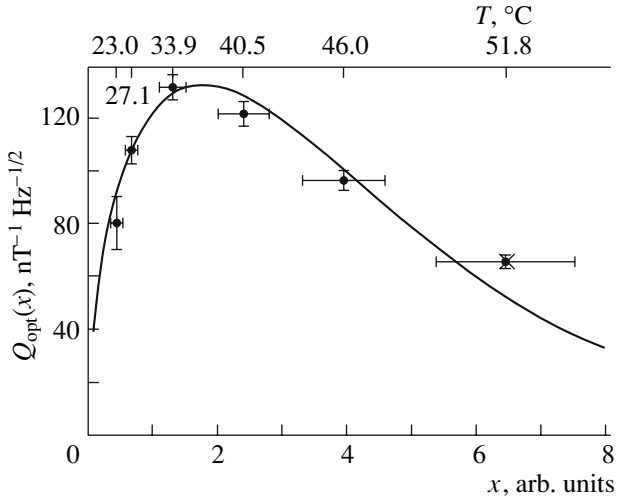

Fig. 7. Dependence of the Q factor in a cesium cell on its optical thickness.

approximation by formula (16)). In complete agreement with formula (16), the maximum of the Q factor is attained at $I = 3 \pm 1$ for all values of the cell temperature.

The results of approximation of $Q(I)$ for various temperature T were used for plotting the dependence of the Q factor of the resonance, optimized from the intensity of photocurrent, on the optical thickness of the cell (Fig. 7). The optimal value of the Q factor of the resonance is attained at temperature $T = 36.1 \pm 1.8$°C, corresponding to optical thickness $x = 1.61 \pm 0.27$ of the cell (in the optimal rf field), which is in good agreement with the theory. The last point on the graph corresponds to a temperature of 51.8°C, at which the change in the parameters of coating becomes noticeable. The optimal optical thickness corresponds to spin-exchange broadening $\Gamma_{coll}/2\pi = 9.6$ Hz, which differs from theoretical estimate $\Gamma_{coll}/2\pi = 12.8$ Hz by ~ 30%. This discrepancy can be attributed to indeterminacy in the spectral composition of pump radiation (it was assumed in estimates that coefficient α for flashlamp pumping is twice as high as for laser pumping) and to the error in determining of the effective internal diameter of the cell.

CONCLUSIONS

Thus, we constructed a simple model for optimizing the parameters of the M_x resonance in a vacuum cell and performed experiments demonstrating the ability of the model to provide qualitative as well as quantitative estimates.

Let us list the main conclusions following from the proposed model.

(1) The limiting Q factor of a quantum M_x discriminator with optical pumping is controlled by quantum atomic noise and the efficiency of optical pumping and detection of M_x resonance.

(2) The inclusion of spin-exchange broadening of the absorption line in the cell does not change the requirements imposed on pumping radiation: pump light must ensure line broadening in the interval from 1.5 to 3.0 as compared with the dark width.

(3) For cells with a long relaxation time on a coating ($\Gamma_0/2\pi \ll 1$ Hz), the optimal resonance width is entirely controlled by the requirements to optical thickness of the cell ($x_{opt} \geq 1.6$), which makes it impossible to take advantage of ultranarrow resonances in magnetometric circuits.

(4) The sensitivity of a quantum discriminator is limited by the Cramér–Rao relation over time intervals shorter than the reciprocal resonance linewidth.

(5) An increase in the cell length in the direction of the pump beam in the magnetometric circuit makes it possible to reduce the total linewidth in proportion to the decrease in the spin-exchange broadening and thus to reduce the shifts and errors of the magnetometer.

ACKNOWLEDGMENTS

The authors are grateful to E.B. Alexandrov for fruitful discussions, S.P. Dmitriev for providing the magnetic screen and assistance in the research, and M.V. Balabas for preparing the cesium cell with coating.

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Translated by N. Wadhwa